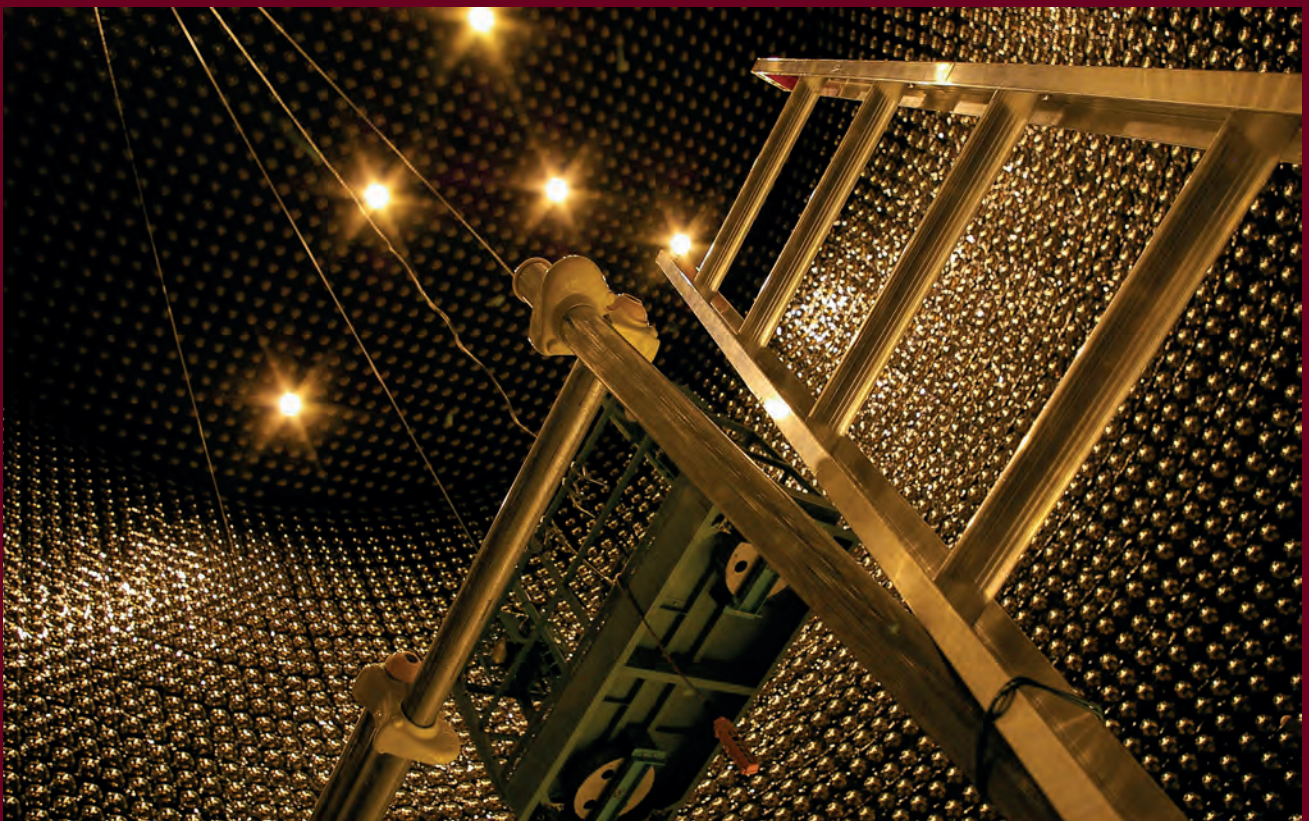


Magdalena Posiadała-Zezula

The Super-Kamiokande detector
and its contributions to neutrino oscillation
searches in the GeV energy range



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View of the interior of the Super-Kamiokande detector, showing the gold-colored photodetectors installed on the detector walls. Photograph taken by Dr Piotr Mijakowski, former member of the Super-Kamiokande collaboration, during the tank opening work in 2006.

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Magdalena Posiadała-Zezula, ORCID 0000-0002-5154-5348, University of Warsaw

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Abstract

This monograph presents studies of neutrino oscillations in the GeV energy range using different sources of neutrinos measured by the Super-Kamiokande (SK) detector: atmospheric neutrinos made by nature and accelerator neutrinos made in the laboratory.

A measurement of the oscillation parameters $\sin^2(\theta_{23})$, Δm_{32}^2 , and δ_{CP} , as well as a determination of the neutrino mass ordering, was performed using 6511.3 live days of atmospheric neutrino data, corresponding to a total exposure of 484 kiloton-years, collected by the Super-Kamiokande detector between 1996 and 2020. This analysis is the first one to include fully contained events from the expanded fiducial volume region and new data collected during 1186 additional live days of operation of SK, after a major detector refurbishment in 2018, which results in 48 % increase in statistics over previous studies. A neutron-tagging technique was employed to statistically separate neutrino and antineutrino interactions, improving sensitivity to the neutrino mass ordering. The results favour the normal mass ordering at the 92.3 % confidence level with constraints on $\sin^2 \theta_{13}$ from reactor neutrino experiments.

In the GeV energy range, Super-Kamiokande also serves as the far detector for the Tokai-to-Kamioka (T2K) long-baseline experiment. The first joint SK+T2K oscillation analysis, based on atmospheric neutrino data from Super-Kamiokande (2008–2020) and accelerator neutrino data from T2K (2010–2020), excluded the CP-conserving value of the Jarlskog invariant with a significance of $1.9\text{--}2.0\sigma$ and showed a slight preference for the normal mass ordering.

The precise event reconstruction, low background levels, and long-term operational stability of Super-Kamiokande have established it as one of the leading detectors in neutrino physics. The detector's design and performance directly shape the development of the next-generation Hyper-Kamiokande detector, which will feature a substantially larger fiducial volume and improved control of systematic uncertainties. Sensitivity studies indicate that Hyper-Kamiokande could achieve a 5σ discovery of leptonic CP violation within three years, assuming maximal CP violation and known mass ordering, or within six years when including its own atmospheric neutrino constraints.

Together, the Super-Kamiokande and Hyper-Kamiokande experiments represent a continuing effort to deepen our understanding of neutrino oscillations and the fundamental properties of neutrinos, paving the way for future discoveries in particle physics.

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Neutrinos

Discovery of neutrinos is strongly correlated with the studies of radioactivity: α (charged helium), β (electrons) and γ (high-energy light), related to radiation. It was believed that α and β spectra emitted in nuclear decays were discrete, which is a natural consequence of the kinematics of two-body decays with conservation of energy and momentum laws. However, in 1930, James Chadwick observed that the energy spectrum of electrons emitted in nuclear β decay was continuous and under the assumption of two-body decay would break the conservation laws [1]. In order to explain these observations, Wolfgang Pauli proceeded to propose a “desperate remedy” which would explain the puzzling situation and allow to save the conservation of energy and momentum laws. He suggested that the missing energy was being carried away by the third particle emitted in the decay. It was Enrico Fermi [2] who three years later called this new particle the neutrino and used it in his theory of weak interactions. It took another 23 years before Frederick Reines and Clyde Cowan made the first experimental observation of neutrinos. Their pioneering experiment for which Reines was awarded the Nobel Prize in 1995 [3] took place at the Savannah River reactor [4]. In the experiment they used the inverse of the β decay (IBD) reaction of reactor antineutrinos $\bar{\nu}_e$:

$$\bar{\nu}_e + p \rightarrow e^+ + n \quad (1.1)$$

The target was water with cadmium chloride CdCl_2 dissolved in it. The positron was detected by its slowing down and annihilating with an electron, producing two 0.5 MeV gamma rays in opposite directions. The neutron was also slowed down and captured by the Cd, microseconds after the positron capture [4]. In this way anti-electron neutrino ($\bar{\nu}_e$) was discovered. Scientists still wondered if they could find more types of these ghostlike particles that pass through everything. Subsequent accelerator experiments confirmed that there are two more distinct types of neutrinos according to the type of charged particles found in the interaction. Thus, in 1962 Leon Lederman, Melvin Schwartz and Jack Steinberger discovered muon neutrinos (ν_μ) with the usage of the accelerator neutrino beam method. As a result of their discovery in 1988 they were awarded Nobel Prize [3]. The last known neutrino - tau neutrino (ν_τ) was detected for the first time in 2000 by the Direct Observation of the Nu Tau (DONuT) experiment at Fermilab [5, 6].

1.1. Neutrinos in the Standard Model

Up to now, experimental discoveries show that neutrinos come in three types: electron, muon and tau neutrinos, (ν_e, ν_μ, ν_τ) called flavors. In the Standard Model of particle physics [7] they belong to the

family of leptons. Three different neutrinos are complemented by antineutrinos ($\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$). Neutrinos are neutral particles which interact with matter only weakly - this allows them to travel very long distances without any disturbances. As an example, the value of the mean free path for neutrinos in water is larger than a light year. For many years we believed that neutrinos are massless particles as introduced in the Standard Model. However, results on neutrino oscillations coming from the last 27 years provide clear evidence that there is a spectrum of masses for neutrinos.

1.2. Three neutrino flavor oscillations in vacuum

Available data on the oscillations of the solar (ν_e), atmospheric (ν_μ and $\bar{\nu}_\mu$), accelerator (ν_μ, ν_e and $\bar{\nu}_\mu, \bar{\nu}_e$) and reactor ($\bar{\nu}_e$) neutrinos can be described with three flavor neutrino mixing. Note that the results obtained by the LSND [8] experiment suggesting transitions to a fourth neutrino state are still not confirmed and we do not discuss them here. Moreover, studies of Z^0 production in $e^+ + e^-$ collisions at LEP give the most actual number of light neutrino types, which is equal to $N_\nu = 2,984 \pm 0,008$ [9]. Oscillations among all three neutrino species can be expressed by 3 x 3 unitary matrix, called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [10, 11]:

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}c_{12}s_{23}e^{i\delta} & c_{23}c_{12} - s_{13}s_{12}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{12}c_{23}e^{i\delta} & -s_{23}c_{12} - s_{13}s_{12}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\gamma} & \\ & & e^{i\eta} \end{pmatrix}$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, δ is the Dirac phase and γ, η are Majorana phases. When CP violation occurs these phases are not equal to zero. However, if neutrinos are Dirac particles, this implies that Majorana phases are zero and we can forget about the right hand side of the given formula. Anyhow, Majorana phases do not affect the oscillation mechanism so for the simplicity we will assume that they vanish. For this situation neutrino mixing matrix U is similar to the CKM quark mixing matrix in terms of three mixing angles and one CP violation phase. The oscillation probability is given by the formula:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \Phi_{ij} + k \cdot 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin 2\Phi_{ij} \quad (1.2)$$

Here α and β are observable neutrino flavors states for e, μ , τ flavors, and:

$$\Phi_{ij} = 1.27 \frac{\Delta m_{ij}^2 [\text{eV}^2] L [\text{km}]}{E_\nu [\text{GeV}]} \quad (1.3)$$

with i and j referring to the mass states. The third term is responsible for CP violation with k equal to 1 for neutrinos and -1 for antineutrinos. For three neutrino mass states, there are three different Δm^2 parameters, although only two are independent. The neutrino mass states ν_1, ν_2 and ν_3 are defined in such a way that the difference between ν_1 and ν_2 represents the smallest splitting. However, the mass of ν_3 relative to ν_1 and ν_2 is arbitrary and the sign of Δm^2 which includes the third mass state may be positive or negative. That is when $m_{\nu_3} > m_{\nu_1}, m_{\nu_2}$ then Δm_{32}^2 will be positive but if $m_{\nu_3} < m_{\nu_1}, m_{\nu_2}$ then Δm_{32}^2 will be negative. The former is called a normal mass ordering (NO) and the latter is the inverted mass ordering (IO). This is currently still one of the open questions to the neutrino physics which ordering is correct.

The PMNS can be split into three factors describing rotations around three mixing angles, θ_{12} , θ_{13} and θ_{23} [12]:

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \\ & & 1 \end{pmatrix}$$

where each group of oscillation parameters (Δm_{ij}^2 and θ_{ij}) may be placed into three given sectors: sector 12, sector 13 and sector 23. Although, in general there will be mixing among all three flavors of neutrinos, two-generation mixing may often be assumed. If the mass scales are quite different (for example $m_3 \gg m_2 \approx m_1$), then the oscillation phenomena tend to decouple and the two-generation mixing model is a good approximation in limited regions. This is the case when we investigate sector 12 often called the solar neutrino sector as well as sector 23 (atmospheric neutrino) as will be shown in the following section. In case of neutrino oscillations for just two flavors, the formalism is simplified. As an example, let us consider two flavors, named as $|\nu_\mu\rangle$ and $|\nu_e\rangle$. In the oscillation mechanism they are treated as linear combinations of two mass eigenstates, denoted as $|\nu_1\rangle$ and $|\nu_2\rangle$, respectively:

$$\begin{pmatrix} |\nu_\mu(t)\rangle \\ |\nu_e(t)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1(t)\rangle \\ |\nu_2(t)\rangle \end{pmatrix}$$

The mass eigenstates propagate in space and for ($\hbar = c = 1$) are given by $|\nu_i(t)\rangle = |\nu_i(t=0)\rangle e^{-iE_i t}$ where index “i” may be equal to 1 or 2, respectively. These two states evolve differently, thus:

$$\begin{aligned} |\nu_\mu(t)\rangle &= \cos\theta e^{-iE_1 t} |\nu_1(t=0)\rangle + \sin\theta e^{-iE_2 t} |\nu_2(t=0)\rangle, \\ |\nu_e(t)\rangle &= -\sin\theta e^{-iE_2 t} |\nu_1(t=0)\rangle + \cos\theta e^{-iE_2 t} |\nu_2(t=0)\rangle \end{aligned} \quad (1.4)$$

A state originally $|\nu_\mu\rangle$ may oscillate into $|\nu_e\rangle$ with the probability:

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(\frac{(E_2 - E_1)t}{2} \right) \quad (1.5)$$

and the probability for it to remain as itself is:

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - P(\nu_\mu \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2 \left(\frac{(E_2 - E_1)t}{2} \right) \quad (1.6)$$

In experiments for which $m_i/p_i \ll 1$ the energy and momentum of two mass eigenstates are very close ($E_i = p + \frac{m_i^2}{2p}$, $t = \frac{L}{c}$), and we can rewrite the term:

$$\frac{(E_2 - E_1)t}{2} = \frac{\Delta m^2 c^4}{4\hbar c} \frac{L}{E} = 1.27 \frac{\Delta m^2 [\text{eV}^2] L [\text{km}]}{E [\text{GeV}]} \quad (1.7)$$

where E is the neutrino energy, and L is the distance that neutrino travels from its source to the detector and $\Delta m^2 = m_2^2 - m_1^2$.

Thus, the expression for the oscillation probability may be rewritten as follows:

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= \sin^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2 L}{E} \right), \\ P(\nu_\mu \rightarrow \nu_\mu) &= 1 - P(\nu_\mu \rightarrow \nu_e) \end{aligned} \quad (1.8)$$

To conclude, the oscillation wavelength depends upon the experimental parameters L (distance neutrino source-detector) and E (neutrino energy) as well as the oscillation parameter Δm^2 . The amplitude of the oscillation is constrained by the mixing term, $\sin^2 2\theta$. Finally, neutrino oscillations are a quantum mechanical consequence of existence of neutrino masses and the small relative difference between them. In other words, if neutrinos were massless particles or the difference between masses were too large, we would not be able to observe oscillation phenomena.

For long-baseline accelerator and atmospheric neutrinos, where neutrino baselines range from tens of kilometers to several thousand kilometers, and typical neutrino energies range from MeV to several GeV, the Δm_{21}^2 terms are sub-dominant, leading to approximate flavor oscillation probabilities:

$$\begin{aligned} P(\nu_\mu \leftrightarrow \nu_e) &\approx \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left(1.27 \frac{\Delta m^2 L}{E} \right), \\ P(\nu_\mu \rightarrow \nu_\mu) &\approx 1 - 4 \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23}) \\ &\quad \times \sin^2 \left(1.27 \frac{\Delta m^2 L}{E} \right), \\ P(\nu_e \rightarrow \nu_e) &\approx 1 - \sin^2 2\theta_{13} \sin^2 \left(1.27 \frac{\Delta m^2 L}{E} \right). \end{aligned} \quad (1.9)$$

The approximate oscillation probabilities in Eq. (1.9) are primarily functions of the mixing angles and the absolute value of the squared-mass difference Δm^2 . The phase δ_{CP} , and the neutrino mass ordering, i.e., the sign of the squared-mass difference, are sub-leading effects which make them challenging experimental signatures.

1.2.1. Matter effects in neutrino oscillations

Neutrino oscillations in matter enhance the dependence of oscillation probabilities on the neutrino mass ordering. In matter, due to an increased forward scattering amplitude, electron-flavor neutrinos experience a larger potential relative to μ and τ flavors which modifies the vacuum Hamiltonian via an additional term,

$$H_{\text{matter}} = H_{\text{vacuum}} + \mathbf{U}^\dagger \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{U}, \quad (1.10)$$

where $a = \pm\sqrt{2}G_{\text{F}}N_e$. Here, G_{F} is the Fermi constant, N_e is the electron density, and $\mathbf{U}$ is the PMNS matrix. The sign of a is positive for neutrinos and negative for antineutrinos. Propagating the neutrino states according to the matter Hamiltonian leads to a matter-effective squared-mass difference and mixing angle:

$$\begin{aligned} \Delta m_{\text{M}}^2 &= \Delta m^2 \sqrt{\sin^2 2\theta_{13} + (\Gamma - \cos 2\theta_{13})^2}, \\ \sin^2 2\theta_{13,\text{M}} &= \frac{\sin^2 2\theta_{13}}{\sin^2 2\theta_{13} + (\Gamma - \cos 2\theta_{13})^2}, \end{aligned} \quad (1.11)$$

where $\Gamma \equiv 2aE/\Delta m^2$.

For constant density matter the resulting oscillation probabilities can be written using effective mixing parameters. The $P(\nu_\mu \rightarrow \nu_e)$ probability from Eq. (1.9) can be rewritten by replacing Δm_{31}^2 and θ_{13} by their matter-effective parameters from Eq. (1.11).

Moreover, Eq. (1.11) shows that the effective quantities depend on the sign of Δm^2 . In particular, for neutrinos in the normal ordering, $\Gamma \approx \cos 2\theta_{13}$ maximizes the effective mixing angle $\sin^2 \theta_{13,\text{M}}$.

A maximum also occurs for antineutrinos in the inverted ordering. This maximum effective mixing angle predicts a resonant enhancement of muon-to-electron flavor conversions for either neutrinos or antineutrinos according to the neutrino mass ordering. This effect will be crucial for atmospheric neutrinos described in Chapter 3.

1.3. Experimental searches for neutrino oscillations

In general, there are two types of oscillation experiments: *disappearance* and *appearance*. Let's consider a pure source of neutrino type x . In a *disappearance* experiment, one looks for a deficit in the expected flux of ν_x . This requires accurate knowledge of the flux, which is often difficult to predict precisely without corresponding hadron production measurements as well as near-far detector design (like T2K experiment - see Chapter 2.2). The near detector measures the flux without oscillations, which is later on used to predict the unoscillated event rate in the far detector. A deficit compared to prediction indicates *disappearance*.

Appearance experiments search for $\nu_x \rightarrow \nu_\beta$ oscillations by directly observing interactions of neutrinos type β .

In our review of the neutrino oscillation experiments we separately describe given sectors of oscillation parameters Δm_{ij}^2 and θ_{ij} , where ij stands for 12, 13, 23 respectively. We review the neutrino experiments, starting from the earliest measurements. As a result, at first sector 12 known as solar sector is investigated. Next sector 23, named atmospheric and accelerator sector, is introduced. Finally the sector 13 is studied by the reactor experiments.

1.3.1. Solar neutrinos - oscillation parameters: Δm_{21}^2 and θ_{12}

Sector 12 is mainly investigated by the neutrinos produced in the Sun, which is the reason why in the literature we may also encounter a name - solar neutrino sector. This field was probed by several experiments using different detection techniques. All projects listed below were/are directly examining solar neutrinos:

- radiochemical detectors: Chlorine (Homestake [13]), Gallium (SAGE [14], GALLEX [15] and its successor GNO [16]),
- water Cherenkov detectors: with H_2O (Kamiokande-II [17] and Super-Kamiokande (SK) [18, 19]) and with D_2O (SNO [20]),
- liquid scintillator detectors: KamLAND and Borexino [21].

In addition to direct solar neutrino projects there was also a very vital contribution to θ_{12} and Δm_{21}^2 measurements coming from the long baseline reactor experiment KamLAND [22].

Solar neutrino flux

The Sun's core emits neutrinos in series of thermonuclear interactions, among which the most significant ones are listed in Table 1.1 while the CNO cycle is included in the Table 1.2.

Reactions from pp-chain (99%) listed in Table 1.1 and CNO cycle (1%) listed in Table 1.2 result in fusion of protons $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e$, where the energy released in the reaction is of the order of $Q = 4m_p - m_{{}^4\text{He}} - 2m_e = 26 \text{ MeV}$ and is mostly radiated by the photons and neutrinos, which carry only $\langle E_\nu \rangle = 0.59 \text{ MeV}$.

Scientists were able to gather all relevant information for the Sun into so-called Standard Solar Model (SSM) with the aim to predict solar neutrino fluxes on Earth. The most reliable model of this type was created in 1962 and developed from then on by John Bahcall and others [23]. The SSM predictions for the neutrino fluxes and spectra are presented in Fig. 1.1. The neutrinos labeled pp,

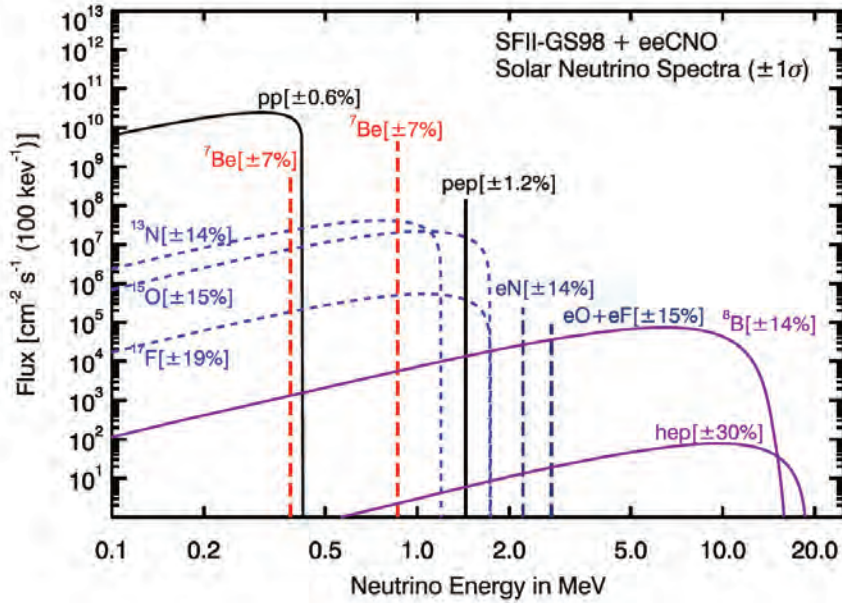
Table 1.1. The most significant thermonuclear reactions from pp chain producing neutrinos in the Sun with their abbreviations.

Reaction	abbreviation
$p + p \rightarrow d + e^+ + \nu_e$	pp
$p + e^- + p \rightarrow d + \nu_e$	pep
${}^3\text{He} + p \rightarrow {}^4\text{He} + e^+ + \nu_e$	Hep
${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e + (\gamma)$	${}^7\text{Be}$
${}^8\text{B} \rightarrow {}^8\text{Be}^* + \nu_e + e^+$	${}^8\text{B}$

Table 1.2. The CNO cycle.

Reaction	abbreviation
${}^{13}\text{N} \rightarrow {}^{12}\text{C} + e^+ + \nu_e$	${}^{13}\text{N}$
${}^{15}\text{O} \rightarrow {}^{15}\text{N} + e^+ + \nu_e$	${}^{15}\text{O}$
${}^{17}\text{F} \rightarrow {}^{17}\text{O} + e^+ + \nu_e$	${}^{17}\text{F}$

pep, ${}^7\text{Be}$, ${}^8\text{B}$ and Hep belong to the pp chain which for a star like the Sun dominates over those from the CNO cycle. Of the neutrinos in the pp chain, those from the initial reaction $p + p \rightarrow d + e^+ + \nu_e$ make up over 92% of the entire solar flux [24]. Flux coming from neutrinos produced in the Sun by electron capture reactions on ${}^{13}\text{N}$, ${}^{15}\text{O}$ and ${}^{17}\text{F}$, to which we refer as ecCNO neutrinos, is also included in Fig. 1.1

**Figure 1.1.** Differential Standard Solar Model neutrino fluxes. In addition to standard fluxes, ecCNO line spectra of neutrinos have been added as well. Electron capture fluxes are given in $\text{cm}^{-2} \text{s}^{-1}$. This figure is taken from [25].

Solar neutrino problem

Different parts of the solar neutrino spectrum have been studied in various experiments. In Table 1.3 a list of all solar experiments is given with additional information. We will give short description of the most important ones.

The first indication of neutrino oscillations came from the Chlorine solar neutrino experiment conducted by Raymond Davis in the Homestake mine in South Dakota in 1968 [13]. It was a radiochemical experiment which detected neutrinos via the reaction:

$$\nu_e + {}^{37}\text{Cl} \rightarrow e^- + {}^{37}\text{Ar} \quad (1.12)$$

Table 1.3. List of past and running solar neutrino experiments.

Experiment	Target material	Mass (ton)	Years
Homestake	C ₂ Cl ₄	615	1970–1994
SAGE	Ga	50	1989–now
GALLEX	GaCl ₃	100	1991–1997
GNO	GaCl ₃	100	1998–2003
Kamiokande	H ₂ O	3.000	1987–1995
Super-Kamiokande	H ₂ O	50.000	1996–now
SNO	D ₂ O	1.000	1999–2006
KamLAND	liquid scintillator	1.000	2001–now
Borexino	liquid scintillator	300	2007–2021

The energy threshold for this reaction is 0.8 MeV, so an experiment of this kind cannot see pp neutrinos (see Fig. 1.1). Produced radioactive atoms of ³⁷Ar have half life time ($\tau_{1/2}$) of 34.8 days, and after time of 2-3 $\tau_{1/2}$ values Davis and other collaborators could extract radioactive products from the tank through chemical methods and count them with the help of proportional counters. The problem which aroused afterwards, was the fact that this experiment observed only a third of the neutrinos coming from the Sun predicted by the SSM. The solar neutrino problem as it came to be known was sustained by the findings of the Gallium experiments GALLEX [15] and GNO [16] at Gran Sasso, Italy and SAGE [14] at Baksan, Russia. The threshold for the reaction was about 0.2 MeV, significantly lower than in the Chlorine case and as a result they were able to see also pp neutrinos. Obtained results, however, still pointed out to decreased neutrino flux in comparison to that predicted by SSM.

Next information on the neutrino flux but this time in the higher energy range ($E_\nu > 5$ MeV) was delivered by the Kamiokande [17] and its successor Super-Kamiokande [18, 19] experiments using a totally different observation technique. Both detectors were located in the Kamioka mine in Japan. They use Cherenkov radiation to detect particles produced in neutrino interactions in water. This detection technique is based on the fact that a particle moving with a speed greater than the speed of light in given medium will emit light in a cone around particle direction, called Cherenkov radiation. Detection of this light by the photomultipliers (PMTs) installed on the walls of the detector enables determination of the particle energy as well as its incoming direction. Super-Kamiokande is the far station in the T2K experiment and a detailed overview of the detector is given in the next Chapter in Section 2.1. Now, for example, when a solar neutrino enters the water tank it may interact via elastic scattering (ES):

$$\text{ES} : \nu_x + e^- \rightarrow \nu_x + e^- \quad (1.13)$$

and what is recorded is the recoil electron while x represents all types of neutrinos, like ν_e , ν_μ and ν_τ . However, the sensitivity to ν_μ and ν_τ is smaller than for the ν_e because of the relation between corresponding cross sections: $\sigma_{(\nu_\mu, \tau + e)} \approx 0.16\sigma_{(\nu_e + e)}$ [26]. The detection thresholds were 7 MeV and 5 MeV for Kamiokande and Super-Kamiokande experiments, respectively. They were able to detect only neutrinos coming from the ⁸B decays (see Table 1.1). The applied detection technique allowed Kamiokande to give for the first time a proof that measured neutrinos do really come from the direction of the Sun. Both these detectors observed again a massive deficit in the number of neutrinos in comparison to SSM. For example, in case of the Super-Kamiokande the relative rate of observed to expected neutrinos was equal to 0.39 ± 0.06 [19].

Still the solar neutrino problem was unsolved and what scientists began to think was that perhaps something is happening to the neutrinos as they come from the Sun to Earth or simply predictions of SSM are not correct. In case of the first point, one should remember that in the Sun only electron neutrinos are produced and detectors on Earth have been sensitive mainly to these electron neutrinos. So if the neutrinos were undergoing a “flavor” oscillation, the probability to detect those electron neutrinos would reduce the observed rate. The solution was provided by the Sudbury Neutrino Observatory (SNO), which was underground heavy water (D₂O) Cherenkov experiment located in the Creighton mine, Ontario in Canada [20, 27]. What makes SNO better than the Super-Kamiokande was the fact that SNO was filled with heavy water (D₂O) which allowed to detect solar neutrinos with three reactions:

$$\text{Charge Current (CC)} : \nu_e + d \rightarrow p + p + e^-$$

$$\text{Neutral Current (NC)} : \nu_x + d \rightarrow p + n + \nu_x$$

$$\text{Elastic Scattering (ES)} : \nu_x + e^- \rightarrow \nu_x + e^-$$

where $x = e, \mu, \tau$. The first reaction with charged current exchange (CC) is sensitive only to ν_e as solar neutrinos energies (a few MeV) allow only to produce electrons in the final state and not μ or τ leptons. Other reactions NC (neutral current exchange) and ES may happen to all three types of neutrinos. The threshold of the NC reaction is 2.2 MeV. In D₂O it was measured by neutron capture in deuterium, which resulted in 6.25 MeV γ -ray. For pure D₂O the capture efficiency was low thus SNO collaboration decided to add 2 tons of salt (NaCl) into the heavy water. Data taking period with enriched D₂O was named Phase-II of the experiment. In the third stage (Phase-III), NaCl was removed and neutron counters were installed in order to provide independent NC measurements with different systematics than in Phase-II. The final results from SNO Phase-II are the following [20]:

$$\Phi_{\text{SNO}}^{\text{CC}} = (1.68 \pm 0.06^{+0.08}_{-0.09}) \times 10^6 \text{cm}^{-2} \text{s}^{-1}$$

$$\Phi_{\text{SNO}}^{\text{ES}} = (2.35 \pm 0.22 \pm 0.15) \times 10^6 \text{cm}^{-2} \text{s}^{-1}$$

$$\Phi_{\text{SNO}}^{\text{NC}} = (4.94 \pm 0.21^{+0.38}_{-0.34}) \times 10^6 \text{cm}^{-2} \text{s}^{-1}$$

Comparison of the presented results with SSM, shows that $\Phi_{\text{SNO}}^{\text{NC}}$ finally agreed with expectations of the SSM. This measurement provided direct evidence for flavor conversion of solar neutrinos of type $\nu_e \rightarrow \nu_{\mu, \tau}$ which was the reason why previous measurements could observe a deficit in solar electron neutrino flux. All the results from SNO Phase-II are presented in Figure 1.2. The illustration shows combined flux of $\nu_{\mu+\tau}$ versus flux of ν_e . In addition, the Super-Kamiokande results for ES [19] are also included with the narrow band parallel to the SNO ES results. For all measured fluxes we observe a region of overlap (see black dot representing the best fit point). Moreover, the $\nu_{\mu+\tau}$ flux was determined for the first time and its contribution to the total solar neutrino flux is very significant. In the best fit point it is almost two times larger than ν_e flux.

Another important contribution to the solar neutrino sector came from the Kamioka Liquid Scintillator Anti-Neutrino Detector (KamLAND) experiment [22]. KamLAND is located at the site of the earlier Kamiokande experiment at a depth of 2700 m water equivalent. The heart of the detector is 1 kton of highly purified liquid scintillator enclosed in a balloon suspended in purified mineral oil. The detector is inside a 18 m diameter stainless steel sphere. An array of 50 cm diameter photomultiplier tubes (PMTs) is mounted on the inner surface of the sphere (around 1900 PMTs). The KamLAND is surrounded by the 55 Japanese nuclear power reactor units, each being an isotropic $\bar{\nu}_e$ source. This $\bar{\nu}_e$ have energies similar to those of solar neutrinos and should therefore be subject to the same kind of oscillation phenomena. In the reaction of interest, which is IBD 1.1: $\bar{\nu}_e + p \rightarrow e^+ + n$, prompt scintillation light is seen from the outgoing positron and light from 2.2 MeV γ ray emitted when the

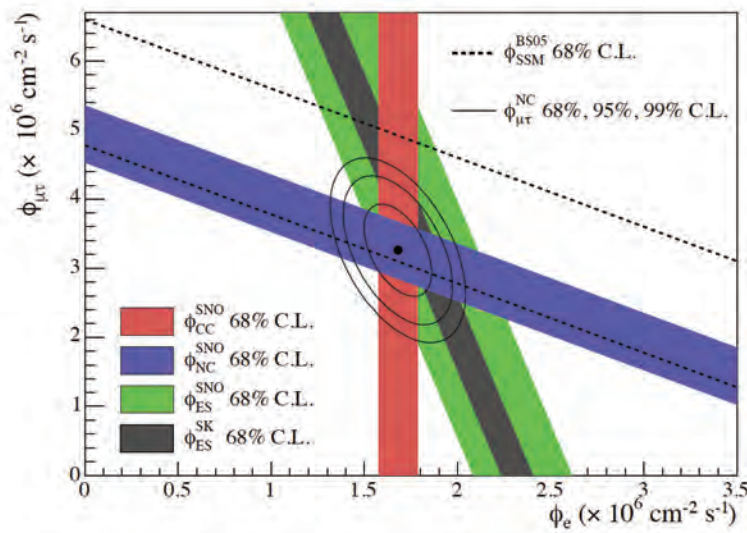


Figure 1.2. SNO results after Phase-II: flux of $\nu_{\mu+\tau}$ versus flux of ν_e ^8B solar neutrinos. Contributions coming from all three reactions: CC (red), NC (blue) and ES (green for SNO and grey for SK) are indicated by the filled bands. The 68% limits on total ^8B solar neutrino flux predicted by the SSM are shown as dashed lines. In addition, the Super-Kamiokande results [19] are also shown with the narrow band parallel to the SNO ES results. Black dot indicates the best fit point together with contours for 68, 95, and 99% CL. Figure is taken from [20]

neutron captured on a proton is seen some 200 μs later. KamLAND was the first experiment to observe reactor electron antineutrinos $\bar{\nu}_e$ with energies of a few MeV from nuclear reactors typically 180 km away [28].

It is also important to mention about another real time solar neutrino experiment, Borexino, placed at Gran Sasso in Italy, which started data taking in 2007 [21]. Detection of solar neutrinos was based on $\nu_e + e$ scattering in a very pure liquid scintillator. The reaction threshold was very low, because only 250 keV, which allowed to measure monochromatic 0.861 MeV ^7Be (see Fig. 1.1) for the first time.

1.3.2. Atmospheric and accelerator neutrinos - oscillation parameters Δm_{32}^2 and θ_{23}

The next sector to be investigated is the one defined by the oscillation parameters θ_{23} and Δm_{32}^2 . This time sector 23 is named the atmospheric sector as evidence suggesting oscillation was coming from atmospheric neutrino observations which resulted in measurements of θ_{23} and Δm_{32}^2 . For the last 20 years we have been measuring these oscillations parameters also by the accelerator experiments in which a neutrino beam is produced in the laboratories. We will start a discussion with atmospheric neutrino experiments and later move to accelerator experiments.

Atmospheric neutrino experiments

Similarly to solar neutrino physics, atmospheric neutrino searches were using several different detection techniques. They are based on:

- water Cherenkov detectors: Kamiokande [29], IMB [30] and Super-Kamiokande [31],
- fine-grained tracking detectors: NUSEX [32], Frejus [33], MACRO [34] and Sudan-2 [35].

Atmospheric neutrinos are produced in the interactions of primary cosmic rays (mainly protons) with the nuclei present in the atmosphere. These interactions generate secondary particles, which include all the hadrons and their decay products. In particular, many secondary pions are produced.

They decay into:

$$\begin{aligned}\pi^+ &\rightarrow \mu^+ + \nu_\mu \\ \pi^- &\rightarrow \mu^- + \bar{\nu}_\mu\end{aligned}$$

The muons, which decay before hitting the ground, generate electrons, electron neutrinos and muon neutrinos through processes:

$$\begin{aligned}\mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_\mu \\ \mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu\end{aligned}$$

The energies of neutrinos produced in such a way range up to TeV.

At low energies ($E \leq 1\text{-}2$ GeV) for which most of the muons decay before hitting the ground, we expect to have two $\nu_\mu + \bar{\nu}_\mu$ components for each produced $\nu_e + \bar{\nu}_e$. The ratio (R):

$$R \equiv \frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} \quad (1.14)$$

increases with the energy as more energetic muons reach the Earth's crust before they decay.

The first experimental observations of atmospheric neutrinos started in 1960 by F. Reines and colleagues [36]. Next, Kamiokande [29] and IMB [30] as well as NUSEX [32] and Frejus [33], were also studying this physics field and the conclusions from the observations were not changed. Similarly to the solar sector, scientists began to call the measured deficit of atmospheric neutrinos as atmospheric neutrino anomaly.

The first direct solution to the problem of the atmospheric neutrino anomaly was presented by the Super-Kamiokande in 1998 [31]. Among the authors list of this famous publication one may find name of Prof. dr hab. Danuta Kielczewska, who was employed at the Faculty of Physics, University of Warsaw. This report also gave the first essential proof for the neutrino oscillation phenomena. Super-Kamiokande collaboration examined μ -like events, which are mostly muon-neutrino and muon antineutrino CC reactions as well as e-like events. They were looking for those events which were fully contained (FC), as well as partially contained (PC). The first sample was required to have a signal in the 22.5 kton fiducial volume of the SK detector (for details of the detector overview, please refer to Section 2.1 on page 25), whereas PC by definition should have also PMTs signals in the outer part of SK. Moreover, FC as well as PC events are classified into single-ring and more complicated multi-ring. Those single-ring events have only one charged lepton which radiates Cherenkov light in the final state, making the particle identification very reliable and clean. In Fig. 1.3, the illustration of the zenith-angle distributions of e-like and μ -like events from the SK measurements [31] is presented. Note that $\cos \theta = 1$ (-1) corresponds to the downward (upward) direction, respectively. These plots are made of single-ring FC events as well as Multi-GeV events (they are visible for $E > 1.33$ GeV). The dashed, black histograms show simulated expectations without an assumption on the neutrino oscillation phenomena, whereas the solid, black lines show the best-fit expectations for the $\nu_\mu \rightarrow \nu_\tau$ oscillations. This oscillation pattern was assumed based on the observations of the e-like and μ -like events. For the first sample we see that both measurements as well as expectations are consistent. On the other hand, the zenith-angle distributions of the μ -like events show a large discrepancy in the expectations for the upward neutrino flux. This fact is explained as follows: muon neutrinos coming from the opposite side of the Earth's atmosphere, travel 10,000 km and oscillate into other neutrino flavors. As we do not see an indication of electron neutrino appearance, this other flavor could be only tau neutrinos.

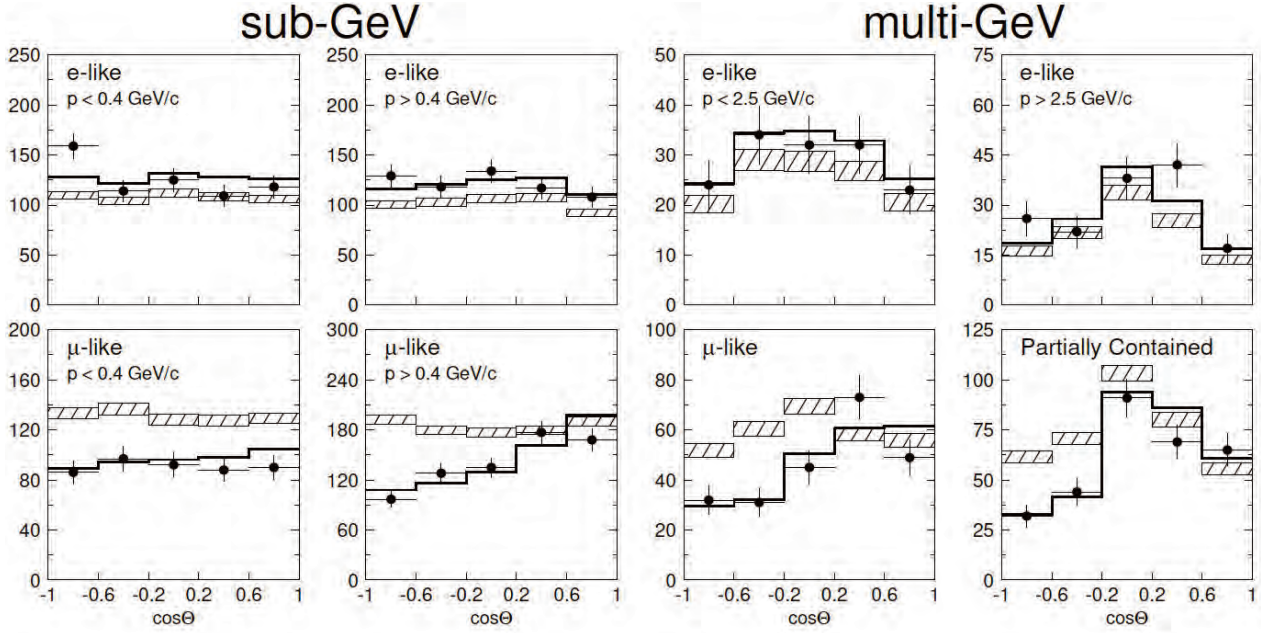


Figure 1.3. First Super-Kamiokande results on the neutrino oscillation phenomena. All plots show zenith-angle distributions for e-like and μ -like events which are grouped into one with visible $E < 1.33$ GeV (Sub-GeV) and $E > 1.33$ GeV (Multi-GeV). The dashed, black histograms represent non-oscillated expectations, while the solid lines stand for the best-fit expectations for the transition of the $\nu_\mu \rightarrow \nu_\tau$. This figure is taken from [31].

Neutrino oscillations with atmospheric neutrinos can also be studied by the neutrino telescopes like: KM3NeT/ORCA [37] or IceCube [38]. KM3NeT/ORCA is a water Cherenkov neutrino detector under construction and anchored at the bottom of the Mediterranean Sea. The first searches of KM3NeT focuses on an initial configuration of ORCA, referred to as ORCA6, which comprises six out of the foreseen 115 detection units of photo-sensors. A high-purity neutrino sample was extracted, corresponding to an exposure of 433 kton-years [37].

IceCube is a detector deployed in the ice in Antarctica at the South Pole. It is located at the depth between 1.45 and 2.45 km. In the bottom center of IceCube there is region with denser PMT spacing called Deep Core, which provides access to neutrinos with energies above approximately 5 GeV. Data taken between 2012 and 2021 (3387 days) were utilized for an atmospheric ν_μ disappearance analysis that studied 150 257 neutrino-candidate events with reconstructed energies between 5 and 100 GeV [38].

Accelerator neutrino experiments

Studies of atmospheric neutrino sector show that the most precise measurements of the oscillation parameters Δm_{32}^2 and θ_{23} were performed by the accelerator experiments. The list of all past, ongoing and future experiments are given in the Table 1.4

The first accelerator-based, long-baseline oscillation experiment was the K2K (KEK-to-Kamioka) project [39]. It was a *disappearance* type of experiment and was designed to provide another confirmation of neutrino oscillation phenomena. K2K used a wide-band focused by horns muon neutrino beam, produced by 12 GeV protons from KEK-PS. The beam was measured 300 m from the target station (near detector) and 250 km away in the Super-Kamiokande. Data were taken from 1999 till November 2004. They observed 112 FC events while $158.1^{+9.2}_{-8.6}$ were expected without oscillations. Next, the second oscillation long-baseline project called MINOS [40] entered the game with a near and far detectors system. This time, neutrinos were produced by the NuMI (Neutrinos at the Main Injector) research complex in Fermilab (Fermi National Accelerator Laboratory), located in Batavia, Illinois,

Table 1.4. List of the past, running and future accelerator long-baseline neutrino experiments. Taken from [26].

Experiment	Beamline	Far Detector	L (km)	E_ν (GeV)	Years
K2K	KEK-PS	Water Cherenkov	250	1.3	1999–2004
MINOS	NuMi	Iron Scintillator	735	3	2005–2013
MINOS+	NuMi	Iron Scintillator	735	7	2013–2016
OPERA	CNGS	Emulsion and Lead	730	17	2008–2012
ICARUS	CNGS	Liquid Argon TPC	730	17	2010–2012
T2K	J-PARC	Water Cherenkov	295	0.6	2010–now
NO ν A	NuMi	Liquid scint. track. calorimeter	810	2	2014–now
Hyper-Kamiokande	J-PARC	Water Cherenkov	295	0.6	2028
DUNE	LBNF	Liquid Argon TPC	1300	2-3	2032?

near Chicago. They were using 120 GeV protons from Fermilab Main Injector. What was different from the K2K experiment was the detection technique used. In MINOS, both near and far stations were made of iron-scintillator tracking calorimeters placed in the magnetic field, which allowed to determine the sign of produced particles due to neutrino interactions. The far detector was located 735 km away from the Fermilab in the Soudan mine. MINOS started data taking in 2005.

We would like to mention another accelerator long-baseline project - OPERA [41]. Its main purpose was to observe the *appearance* transition of a type $\nu_\mu \rightarrow \nu_\tau$ by identifying τ leptons with the help of emulsion technique. OPERA was using CNGS (CERN Neutrinos to Gran Sasso) ν_μ beam with average energy of 17 GeV produced by CERN SPS. OPERA had only a far detector placed 730 km away from the neutrino source in the Gran Sasso Underground Laboratory in Italy. The OPERA collaboration managed to detect five τ leptons produced in charged current ν_τ interaction [42].

The first generation of long-baseline experiments listed so far, has confirmed the existence of neutrino oscillation. However, it was the second-generation experiments, whose main goal was to observe the *appearance* of ν_e from the ν_μ neutrino beam ($\nu_\mu \rightarrow \nu_e$). Only by using the *appearance* mode, and by comparison of oscillation probabilities for both neutrino and antineutrino beam modes, it was possible to search for charge-conjugation parity-reversal (CP) symmetry violation in the lepton sector and measurement of the neutrino mass ordering and the octant of θ_{23} . The Tokai-2-Kamioka (T2K) experiment started taking data in 2010 using a newly constructed high-intensity proton beam synchrotron at J-PARC and the Super-Kamiokande as the far detector. The author of this report has been a member of the T2K experiment already for 17 years, starting from the very beginning in 2008. This work describes the most recent results coming from the T2K searches related to neutrino oscillations. However, only the most important observations achieved by the T2K during these several years are included below. In 2011, our paper reported the first indication of a three-flavor neutrino oscillation of a type $\nu_\mu \rightarrow \nu_e$ within a statistical significance of 2.5σ [43]. The observation of six single ring e-like events in T2K exceeded the expectation of a three-flavor neutrino oscillation scenario with a hypothesis of $\sin^2 2\theta_{13} = 0$. Later, in 2013 we finally **observed** the $\nu_\mu \rightarrow \nu_e$ oscillations [44]. Overall, 28 electron neutrino events were detected with an energy distribution consistent with an appearance signal, corresponding to a significance of 7.3σ when compared to 4.92 ± 0.55 expected background events. Only after that, knowing that θ_{13} has non zero value, confirmed also by reactor experiments (see next Section 1.3.3), it was possible to study the CP violation by studying the neutrino and antineutrino data in T2K. In 2020, we reported the first hint of CP violation in NATURE paper [45] with a close to 3σ level and the most recent values in [46] are still below 3σ level.

Another current player in the world is a NO ν A experiment which is using the upgraded NuMI beamline. The far detector of NO ν A is located near Ash River, Minnesota, 810 km away from the neutrino source. The neutrino average energy at the far detector is equal to 2 GeV. Both far and near detectors are tracking calorimeters consisting of planes of polyvinyl chloride cells arranged in vertical and horizontal configurations filled with liquid scintillators. The first physics run in NO ν A has started in 2014, and the project has observed the *appearance* mode $\nu_\mu \rightarrow \nu_e$ relatively fast [47]. In 2019 NO ν A has reported the first observation of the *appearance* of $\bar{\nu}_e$ in the $\bar{\nu}_\mu$ beam [48]. Both experiments: T2K and NO ν A have decided to combine the effort in the neutrino oscillation searches by providing results from a joint analysis combining the 2020 datasets from both experiments into a unified framework using detailed likelihoods and consistent statistical treatment. At the time of publishing this monograph T2K+NO ν A have published their first joint neutrino oscillation results in NATURE paper [49]. The combination of the differing sensitivities to neutrino oscillation for both experiments, together with the T2K+NO ν A data, constrain the oscillation parameters better than either experiment alone.

Currently, there are two future long-baseline experiments under construction: Hyper-Kamiokande [50, 51] and DUNE [52]. The Hyper-Kamiokande detector in Japan will be about 8–10 times larger than Super-Kamiokande.

Author of this report is also a member of the Hyper-Kamiokande experiment, and details of the construction, physics potential of the Hyper-Kamiokande will be discussed in the last chapter.

Deep Underground Neutrino Experiment - DUNE will have a 1300 km long baseline, and the whole project is located in the USA [52]. The DUNE far detector will consist of four modules with at least 10 kt fiducial mass liquid argon Time Projection Chambers (TPC), located at the Stanford Underground Research Facility in South Dakota. The beamline for DUNE will start with 1.2 MW and later it will be upgraded to 2.4 MW. Both future long-baseline neutrino oscillation experiments will have very complementary physics program.

1.3.3. Reactor antineutrinos - oscillation parameter θ_{13}

Reactor antineutrinos are produced through the nuclear fission of heavy isotopes, like ^{235}U , ^{238}U , ^{239}Pu and ^{241}Pu . The reactor neutrinos are mainly produced through the β^- decays of the neutron-rich fission daughters of these four isotopes, in which a bound neutron is converted into a proton while producing an electron and an electron antineutrino. Nuclear reactors are thus a very intense source of $\bar{\nu}_e$ in the MeV energy range. The $\bar{\nu}_e$ reactor flux can be derived using information on thermal power and fuel composition as a function of time. An average of 6 $\bar{\nu}_e$ were produced per fission, leading to 2×10^{20} $\bar{\nu}_e$ emitted every second isotropically for each GW of thermal power. The more comprehensive discussion on the reactor $\bar{\nu}_e$ flux may be found in [26]. We will review here all the reactor experiments, given in the Table 1.5, which contributed to the searches of neutrino oscillations and determination of θ_{13} .

Table 1.5. List of past, running and future reactor oscillation neutrino experiments. Taken from [26].

Experiment	Reactor power (GW)	Detector mass (t)	Baseline (km)	Years
KamLAND	many	1.000	180	2001–now
Double Chooz	4.25×2	8.3	1.05	2011–2018
Daya Bay	2.9×6	20×4	1.65	2011–2020
RENO	2.8×6	16	1.38	2011–now
JUNO	26.6	20 000	53	2025

The measurement of the last remaining mixing angle θ_{13} was a huge desire before 2012. As one of the fundamental parameters describing the properties of neutrinos, θ_{13} was also highly relevant for the phenomenon of CP-violation in the neutrino sector. The importance of the last unknown mixing angle θ_{13} led to a worldwide effort to measure it in high-precision experiments.

The reactor neutrino experiments with a short baseline of 1 km were able to make very precise measurements of $\sin^2 2\theta_{13}$ from the *disappearance* of the $\bar{\nu}_e$. Several methods can potentially be used to detect reactor neutrinos. The first and the most important one is the IBD reaction (1.1) which was used by Frederick Reines and Clyde Cowan to make first experimental observation of neutrinos. The energy threshold of this process is about 1.8 MeV. The IBD event is indicated by a pair of coincident signals consisting of: a prompt signal induced by the positron ionization and annihilation inside the detector, and a delayed signal produced by the neutron captured on a proton or a nucleus (such as Gd). Because of time correlation, IBD can be clearly distinguished from radioactive backgrounds, which usually contain no delayed signal. The energy of the prompt signal is related to the neutrino energy via $E_\nu \simeq E_{\text{prompt}} + 0.78 \text{ MeV} + T_n$, with T_n being the kinetic energy of the recoil neutron. Since T_n , of the order of tens of keV, is much smaller than that of E_ν , the neutrino energy can be accurately determined by the prompt energy, which is crucial for measuring neutrino oscillation. If a neutron is captured on a proton, the delayed signal comes from a single 2.2 MeV γ ray. In comparison, for a neutron captured on Gd, the delayed signal consists of a few γ rays having the total energy of $\simeq 8 \text{ MeV}$. For example, the Daya Bay, Double Chooz, and RENO experiments all utilized Gd-doped liquid scintillators as the medium to detect IBD events. As discussed earlier, the coincidence between the prompt signal and the 8 MeV neutron Gd-capture delayed signal provides a powerful means for identifying IBD events and rejecting accidental backgrounds.

The first results of all three experiments were published in 2012. Double Chooz showed an indication of reactor antineutrino *disappearance* with the ratio of observed to expected events $R = 0.944 \pm 0.016 \pm 0.04$ excluding the no oscillation scenario at the 94.6%CL [53]. Daya Bay observed $R = 0.940 \pm 0.011 \pm 0.004$ corresponding to 5.2σ significance of a non-zero value θ_{13} [54]. RENO published result on $R = 0.920 \pm 0.009 \pm 0.014$ with significance of 4.9σ [55].

The future reactor experiment JUNO, which is located in China, started data taking in the autumn of 2025. It has a baseline of $\sim 50 \text{ km}$ and will use 20 kt liquid scintillator detector [56] to catch neutrinos. The primary goal of the JUNO experiment is to measure the neutrino mass ordering. It will also measure neutrino oscillation mixing parameters with higher precision.

1.4. What do we know, what is still missing?

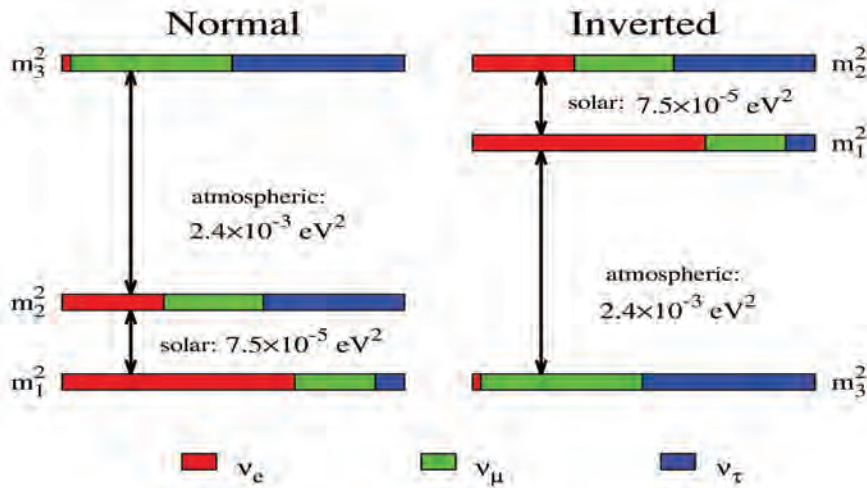
Neutrino oscillation experiments, described in the sections above, deliver very crucial information on the neutrino mixing parameters over the last several years. The summary of the experiments and their contribution to specific neutrino oscillation parameters is given in the Table 1.6. The second (third) column includes the name of the dominant (important) parameter in the 3ν flavor oscillation picture.

Further determination of the neutrino mixing parameters is obtained from the global analyses performed by the phenomenological groups. The most recent results are presented in the Table 1.7 based on work summarized in [57]. There are two possible neutrino mass orderings: Normal Ordering (NO) with condition of $m_1 < m_2 < m_3$ and Inverted Mass Ordering (IO) with $m_3 < m_1 < m_2$. They are graphically introduced in Fig. 1.4. We still don't know which neutrino mass ordering is correct, so we determine independently the neutrino oscillation parameters under the assumption of given ordering.

As we can see from Table 1.7, despite being the smallest among the three neutrino mixing angles in the standard three-neutrino paradigm, θ_{13} is nevertheless the most precisely determined to date

Table 1.6. List of all neutrino experiments, which contribute to determination of neutrino mixing parameters. Taken from [26].

Experiment	Dominant parameter	Important parameter
Solar experiments	θ_{12}	$\Delta m_{21}^2, \theta_{13}$
Reactor long baseline (KamLAND)	Δm_{21}^2	θ_{12}, θ_{13}
Reactor middle baseline: (Daya Bay, RENO, D.Chooz)	$\theta_{13}, \Delta m_{31,32}^2$	
Atmospheric exp. (SK, IceCube Deep Core)		$\theta_{23}, \Delta m_{31,32}^2 , \theta_{13}, \delta_{CP}$
Accelerator long baseline $\nu_\mu, \bar{\nu}_\mu$ disappearance (K2K, MINOS, T2K, NO ν A)	$ \Delta m_{31,32}^2 , \theta_{23}$	
Accelerator long baseline $\nu_e, \bar{\nu}_e$ appearance (MINOS, T2K, NO ν A)	δ_{CP}	θ_{23}, θ_{13}

**Figure 1.4.** The two possible schemes for the neutrino mass orderings: (left panel) the normal ordering, and (right panel) the inverted ordering. The different colors represent the flavor compositions of all the three neutrino mass eigenvalues. Figure is taken from [58].**Table 1.7.** Values of neutrino mixing parameters from one of the global 3ν oscillation analyses [57]. NO corresponds to Normal Ordering of neutrino masses, while IO stands for Inverted Ordering.

ν mixing parameter	Best fit value for NO	Best fit value for IO
$\theta_{12} [^\circ]$	$33.41^{+0.75}_{-0.72}$	$33.41^{+0.75}_{-0.72}$
$\theta_{23} [^\circ]$	$49.1^{+1.0}_{-1.3}$	$49.5^{+0.9}_{-1.2}$
$\theta_{13} [^\circ]$	$8.54^{+0.11}_{-0.12}$	$8.57^{+0.12}_{-0.11}$
$\delta_{CP} [^\circ]$	197^{+42}_{-25}	286^{+27}_{-32}
$\Delta m_{21}^2 \frac{1}{10^{-5} \text{ eV}^2}$	$7.41^{+0.21}_{-0.20}$	$7.41^{+0.21}_{-0.20}$
$\Delta m_{32}^2 \frac{1}{10^{-3} \text{ eV}^2}$	$2.437^{+0.028}_{-0.027}$	$-2.498^{+0.032}_{-0.025}$

neutrino oscillation parameter. What is still unknown in the 3ν flavor oscillation picture is the octant of θ_{23} and the determination of the leptonic CP phase δ_{CP} . The best fit for the complex phase is in NO at $\delta_{\text{CP}} \sim 200^\circ$. CP is conserved when $\delta_{\text{CP}} = 180^\circ$, and current analyses allow it at the confidence level CL of the order of $1-2\sigma$. Future long baseline neutrino oscillation experiments like: Hyper-Kamiokande and DUNE, have a very challenging physics program and will try to measure all remaining parameters in the neutrino oscillations.

Experimental set-up

In this chapter we will describe the experimental features of the Super-Kamiokande detector and a long baseline facility used for the Tokai-2-Kamioka (T2K) project.

2.1. The Super-Kamiokande experiment

Super-Kamiokande (SK) is a 50 kiloton cylindrical world's largest water Cherenkov experiment located within the Kamioka mine in Gifu, Japan [59]. Construction of the detector began in 1991, and five-years later, SK began observations of neutrinos (starting April 1st, 1996). The SK is operated by an international collaboration of about 230 researchers from approximately 50 institutes in Japan, the United States, Poland, Korea, China, Spain, Canada, the UK, Italy, France, and Vietnam. Super-Kamiokande detector is multipurpose machine, in which we are studying the atmospheric, solar and man-made neutrinos, how they oscillate in matter and what their basic properties are. SK has also a broad physics program related to observations of the Universe in different neutrino sources like:

- by measuring the solar neutrinos from the Sun, we can learn how our star works;
- searching for neutrinos from supernova explosions, if detected, may provide a crucial input for understanding the details of the evolution of stars;
- searching for a neutrino signal coming from all past supernova explosions (so called Diffuse Supernovae Neutrino Background – DSNB) which, if detected, may help us to understand the Universe history.

Last but not least, SK important physics goal is to search for the proton and nucleon decay.

SK has already many successes related to the detection of neutrinos from different sources. As it was discussed in the previous Chapter, SK was able to:

- discover neutrino oscillations from the observation of atmospheric neutrinos (1998) [31]
- confirm solar neutrino oscillations from the measurements of solar neutrinos (2001) [18]
- discover the *appearance* of $\nu_\mu \rightarrow \nu_e$ in the accelerator neutrino beam produced at the T2K experiment, for which SK is the far detector (2013) [44]

2.1.1. The Super-Kamiokande detector

The geometry of the SK detector consists of two major volumes, an inner and an outer detector with a cylindrical stainless steel structure placed between them. A schematic view of the SK is presented in Fig. 2.1. Inner detector (ID) is a cylindrical space 33.8 m in diameter and 36.2 m in height. It is equipped with 11,129 inward-facing 50 cm photomultiplier tubes (PMTs). Enclosing the ID is the

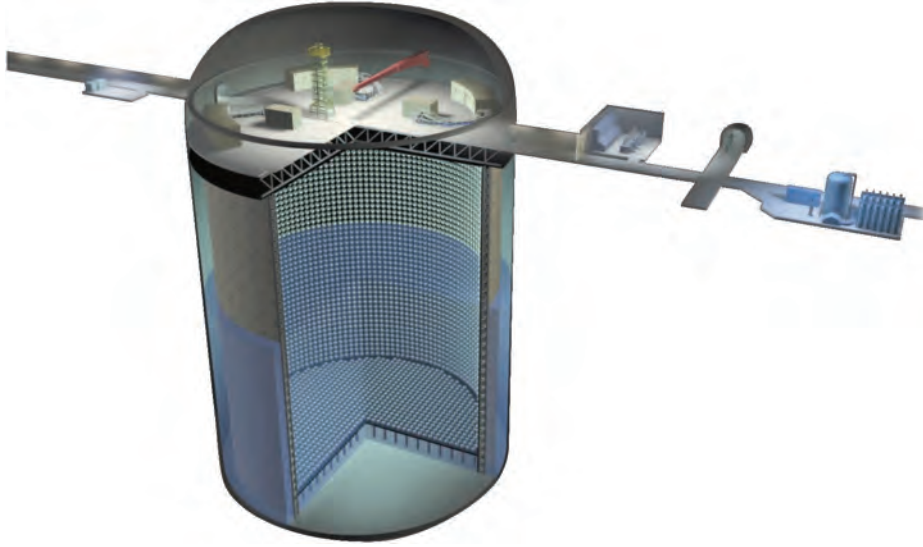


Figure 2.1. Schematic view of the SK detector. The inner and outer detectors are also shown. Figure taken from [60]

outer detector (OD) defined as a cylindrical space of 2 m thick radially on the axis at both ends. The OD contains on its walls 1,885 20 cm diameter PMTs. The PMTs detect the Cherenkov light produced by relativistic charged products of neutrino interactions.

SK operating phases

SK has been operational since its construction in 1996, and, until July 2020, was working with ultra pure water as a target for neutrino interactions. During this period, SK has gone through five data-taking periods, SK-I-V, which were running with very similar operating conditions with a few notable exceptions (see Table 2.1). Starting from the end of SK-I phase (1996–2001), when the implosion of a single PMT occurred during the re-filling of the detector. It created a shockwave and chain reaction that destroyed 6,665 ID PMTs and 1,027 OD PMTs. The detector was rebuilt in the next year with

Table 2.1. Summary of the Super-Kamiokande data-taking periods. An electronics upgrade at the start of SK-IV enabled neutron tagging on hydrogen (H), utilized in the SK-IV and SK-V phases. During 2020, gadolinium sulfate (Gd) was added to the detector’s water to increase the neutron-tagging efficiency.

Phase	Dates	Livetime (Days)	Neutron tagging
SK-I	1996–2001	1489.2	–
SK-II	2002–2005	798.6	–
SK-III	2006–2008	518.1	–
SK-IV	2008–2018	3244.4	H
SK-V	2019–2020	461.0	H
SK-VI	2020–2022	564.4	Gd water (0.01%)
SK-VII	2022–2024	400.0	Gd water (0.03%)
SK-VIII	2024–present		Gd water (0.03%)



Figure 2.2. The author of this monograph visited the SK detector site several times, but inside the ID of SK was so far only once, in October 2018 during tank open works (this was the last time up to now when the SK tank was opened): (*left panel*) in special white uniform, helmet and shoes standing at the top of the SK tank (*right panel*) inside the SK tank standing on a special elevator – wall of the inner detector equipped with PMTs is seen.

almost half of the photocathode coverage (from 40% up to 19%). We had 5,137 PMTs in ID volume and a full complement of OD PMTs for the SK–II period (2002–2005). As a consequence of the accident, all ID PMTs have added plastic shells with 1.0 cm thick acrylic covers, to protect PMTs from another chain reaction. New PMTs were installed to restore photocoverage to original conditions starting with the SK–III phase (2006–2008). At the start of the SK–IV period in 2008, the front-end electronics were upgraded, to a system with ASIC based on a high-speed charge-to-time converter [61]. The SK–IV electronics upgrade extended the window of hit times recorded following neutrino-like events which improved the efficiency for detecting decay electrons and enabled the detection of the 2.2 MeV gamma emission following neutron captures on hydrogen (see Section 3.5.1 for details).

The author of this monograph was inside the SK tank ID and saw all the golden PMTs, once the water was almost gone in October 2018 when refurbishment works inside SK were conducted (see Fig. 2.2).

During 2018, the SK detector was drained to conduct work in preparation for loading gadolinium sulfate, a compound with a high neutron capture cross section, into the detector’s water. The work aimed for several targets: the installation of a new water circulation system, which was able to purify gadolinium sulfate from water, replacement of several hundred malfunctioning PMTs or works related to preventing any leaks. During the next phase, SK–V (2019–2020), the experiment resumed pure-water data taking and once again demonstrated the detector’s stability and performance after the refurbishment. To improve SK neutron detection efficiency and to thereby increase its sensitivity to the diffuse supernova neutrino background flux, 13 tons of (gadolinium sulfate- $\text{Gd}_2(\text{SO}_4)_3$) was dissolved into the detector’s ultrapure water from July 14 to August 17, 2020, marking the start of the SK-Gd phase of operations [62]. The second loading took place in 2022 [63]. Currently, we operate with SK–VIII phase with a concentration of Gd 0.03 %.

50 cm diameter PMTs

The 50 cm diameter PMT (Hamamatsu R3600) used in the ID was developed for SK [64]. Fig. 2.3 shows a schematic view of the PMT. Its dynode structure is the Venetian-blind type.



Figure 2.3. Picture of a 50 cm PMT. Figure taken from <https://www.hamamatsu.com/eu/en/why-hamamatsu/exploring-neutrinos.html>.

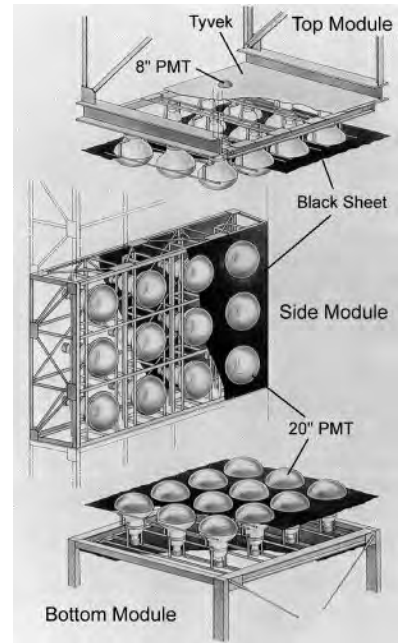


Figure 2.4. A schematic view of the PMT supporting structure and the surrounding components. Figure taken from [59].

A schematic view of the PMT supporting structure and the surrounding components is demonstrated in Fig. 2.4.

The bialkali (Sb-K-Cs) photocathode shows a quantum efficiency characteristic in Fig. 2.5 (*left panel*), with the peak value of 21% at a wavelength of 360–400 nm. Timing resolution (transit time

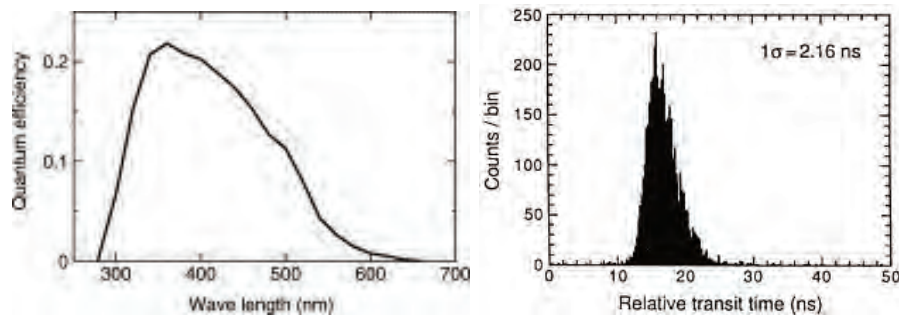


Figure 2.5. Quantum efficiency of the 50 cm PMT as a function of wavelength (*left panel*). Relative transit time distribution for a typical PMT tested with 410 nm wavelength light at the single photoelectron intensity level (*right panel*). Figures taken from [59].

spread) for a 1 p.e. pulse is 2.2 ns as demonstrated in Fig. 2.5 (*right panel*). The dark noise rate at a threshold of 0.25 p.e. adopted in SK is around 6–7 kHz on average in the SK–VI period. From SK–II, after the chain implosion accident, ID PMTs are covered in fiber-reinforced plastic (FRP) cases with acrylic front windows to prevent further accidents.

Cherenkov radiation

The charged particles are measured in the SK detector by Cherenkov light detection. When a charged particle travels through the medium with a speed faster than the light speed in the medium, an electromagnetic radiation named Cherenkov radiation is emitted [65]. The momentum thresholds of the Cherenkov radiation in pure water for charged particles frequently observed in the SK can be evaluated using the formula $p_{\text{th}} \simeq 1.13m_i \cdot c$, where m_i is the particle mass and c is the speed of light (see Table 2.2).

Table 2.2. Momentum thresholds p_{th} (MeV/c) needed for charged particles to emit the Cherenkov radiation in water.

Particle type:	$e^\pm$	$\mu^\pm$	proton
Momentum p_{th} (MeV/c):	0.57	118	1056

The light projected onto the PMT-lined walls of the detector forms ring patterns of hit PMTs. The ring patterns are reconstructed using, for example, the APFIT [66] algorithm for the atmospheric neutrino results, as described in more detail in Section 3.3 on page 54. Photomultiplier timing information is used to reconstruct the initial interaction vertex, after correction for the photon time of flight. A fit considering the spatial distribution and observed charges of hit PMTs determines the particle’s direction, momentum, and particle type. The reconstruction algorithm separates rings into e-like and μ -like: Electrons and photons scatter during their flight in the SK detector. Moreover, they produce electromagnetic showers, resulting in many overlapping rings which appear as a single ring with fuzzy, rough edges. Muons and charged pions, on the other hand, do not form showers and therefore they have sharp ring edges. Higher-momentum particles produce more light, so the charge contained within the ring provides an estimate of the particle’s momentum. Since the neutrino is not directly visible in the SK detector, its energy and direction variables, which are crucial for any oscillation analyses described in the next chapters, are derived from the information we collect on the daughter particles produced by the neutrino interactions. In addition to Cherenkov rings, SK identifies electrons from muon decays $\mu^+(\mu^-) \rightarrow e^+(e^-) + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$. Decay electrons are found by scanning for time-clustered hits above background following a primary neutrino interaction trigger. A hit-time-based fitter estimates the decay electron vertex for each candidate hit cluster, and candidates are accepted if there are 50 or more hits within a 50 ns time window. The overall decay electron tagging efficiency is estimated to be 96 % for μ^+ and 80 % for μ^- in the SK–IV and SK–V periods. The reduced efficiency for μ^- is due to μ^- capture in the water, in which no decay electron is produced.

2.1.2. Gained technical experience in working on Super-Kamiokande detector

In this section, the author discusses the research areas she pursued in Super-Kamiokande collaboration with her students and postdoctoral researchers. The topics can be divided into:

- Calibration procedures of the SK
- Importance of the geomagnetic compensation coils at the SK

Calibration of the SK detector is greatly represented in [67], [68], here the author of this monograph describes only the parts of this complex process in which she was involved together with the group of younger colleagues for whom she is a supervisor and leader.

Charge and timing calibration of the PMTs

The 50 cm in diameter PMTs developed by Hamamatsu Photonics K.K. (R3600-05(A)) [64] are used in the inner detector. These PMTs have a photo-cathode made of bialkali (Sb-K-Cs), and have its maximal photon conversion probability in the wavelength range of Cherenkov light. The timing behavior of PMTs depends on the charge of the measured pulse. In the definition for the PMT charge calibration several names should be explained such as: gain is a conversion factor from the number of photoelectrons to charge (in units of pC), and QE is the product of the quantum efficiency and collection efficiency of photoelectrons onto the first dynode of the PMT. Low-energy physics events like solar neutrinos largely consist of single-photoelectron (single-pe) hits and rely heavily on the QE calibration for their interpretation, whereas high-energy events like those involving TeV-scale muons depend more on proper gain calibration. Knowledge of both gain and QE is important and must be available on a PMT-by-PMT basis.

The timing response calibration of each readout channel including PMT response, cable length, and electronics response, is crucial for the precise reconstruction of the event vertex and directions. Especially, the time-walk effect, where the rise time of a large pulse is faster than that of a smaller pulse, should be calibrated using various intensities of light. The response time of readout channels can vary due to differences in the transition time of PMTs, the lengths of PMT signal cables, and the processing time of readout electronics. The SK uses a nitrogen laser as a light source for timing calibrations. Timing calibrations for SK ID readout channels were conducted based on timing versus pulse height (charge), correlation tables that are called “TQ distributions”. Fig. 2.6 shows a scatter plot of the TQ distribution for one readout channel. The calibration constants, called the “TQ-map”, are derived by fitting the TQ distribution to polynomial functions.

For laser events, a timing correction is applied, called time-of-flight (TOF) correction, which subtracts time of flight from the diffuser ball to the respective PMT position using the group velocity of light with the measured wavelength, $\lambda = 398$ nm. The selected laser hits of each readout channel are divided into 180 bins of charge, called “Qbin”s. Each Qbin is defined as the amount of charge from the PMT in pC; they are defined on a linear scale from 0 to 10 pC (0.2 pC/Qbin) and on a logarithmic scale from 10 to 3981 pC ($50 \log(\text{pC})/\text{Qbin}$). After the TQ distributions are divided into 180 Qbins, the timing distribution is smoothed by a Gaussian to minimize statistical fluctuations. The timing resolution of the SK detector is evaluated using the same data set as used for the TQ-map evaluation.

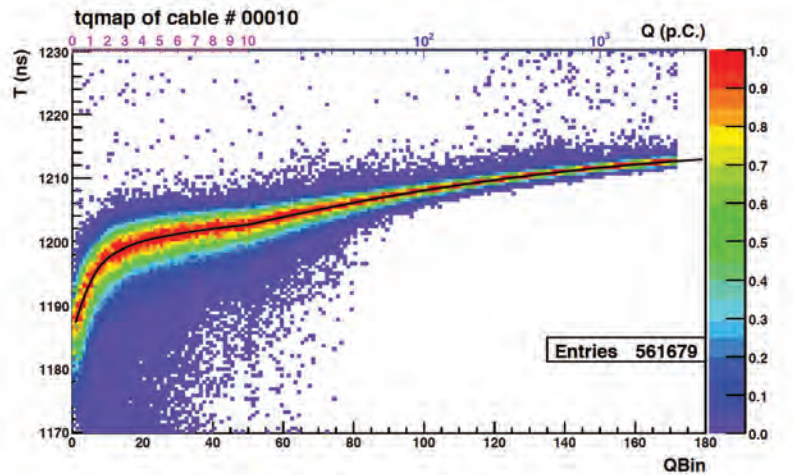


Figure 2.6. Time (T) and charge (Q) distribution for a readout channel. The horizontal axis is the charge (Qbin) of each hit, and the vertical axis is time-of-flight-corrected timing (T) of the hits. Larger (smaller) T corresponds to earlier (later) hits in this figure. Figure is taken from [67].

Studies on reflectivity at the PMTs photocathode surface

These studies are part of the broader calibration step, where light absorption and scattering in detector water are also measured. For this purpose, a collimated laser beam is injected into the tank from the top part, as shown in Fig. 2.7 and the hit timing distribution is compared with MC in some PMT groups.

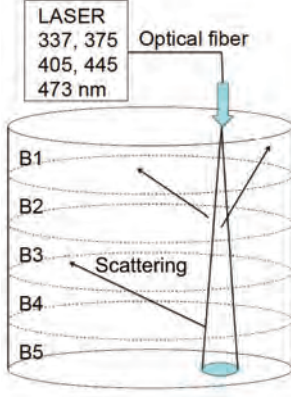


Figure 2.7. Real-time laser system for measurements of absorption and scattering of the Cherenkov light in water and reflectivity at the PMT surface. Analysis was performed using PMTs belonging to five divisions of the barrel region, B1 to B5, and top. Figure is taken from [67]

Light reflection at the PMT photocathode surface is tuned using the same laser data used to determine the water quality parameters. Four layers of material from the surface to the inside of the PMT are taken into account: water (with refractive index $n=1.33$), glass ($n=1.472+3670/\lambda^2$, where λ is the wavelength of the laser), bialkali ($n_{\text{real}} + i \cdot n_{\text{img}}$), and vacuum ($n=1.0$). Here, n_{real} and n_{img} are the real and imaginary parts of the complex refractive index, and an appropriate thickness of the photocathode was based on information from the manufacturer. In the calibration procedure one needs to determine the values of $n_{\text{real}} + i \cdot n_{\text{img}}$ for photocathode assuming different laser wavelengths: $\lambda = 337, 375, 405, 445$ and 473 nm.

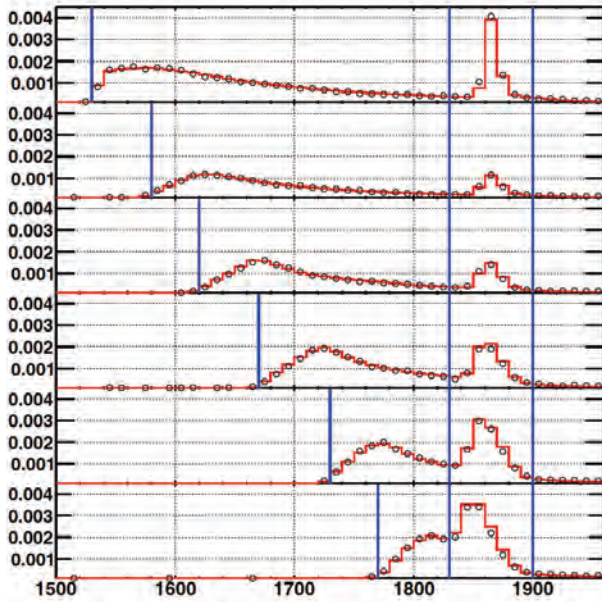


Figure 2.8. Typical TOF-subtracted timing distributions for the vertical down-going laser beam with the wavelength at 405 nm between data (black circle) and MC (red histogram, the best tune). The top panel is for the PMTs at the SK tank top wall. The second to bottom panels correspond to the five barrel wall regions from top to bottom of the SK tank as indicated in Fig. 2.7. Figure is taken from [67]

Fig. 2.8 shows typical hit timing distributions in calibration data and MC after the time of flight of the light from the laser target at the bottom part to the hit PMT position is subtracted. The early broad peaks are from photons scattered in water, while the sharp peaks are due to photons reflected at the detector's bottom.

Studies on geomagnetic field compensation coils inside the tank

The PMTs in SK are larger than usually used PMTs and the electron trajectory distance is longer, which makes the PMT performance more sensitive to the ambient geomagnetic field. The ID PMTs are designed to preserve the qualities in a 100 mG external magnetic field. Thus, the average geomagnetic field of around 450 mG at the detector site could deteriorate the collection efficiency and timing resolution of the PMTs largely and systematically. To compensate for the geomagnetic field, 26 sets of horizontal and vertical Helmholtz coils are arranged along the inner surface of the tank (see Fig. 2.9). The observed residual magnetic field strength after the compensation is 33 mG on average and is less than 100 mG. If all the compensation coils are operational we almost have no magnetic field inside the ID of SK. This was true until the end of 2023, when some of the coil cables failed in three different locations. These locations include:

- V06 disconnected on October 25, 2023. The damaged sub-cable was bypassed on December 1, 2023. It was recovered.
- V01 disconnected on November 15, 2023. V01 was bypassed from the power supply on December 2, 2023.
- H09 disconnected on December 8, 2023. Due to power supply configuration, H06, H07, and H08 were also off.

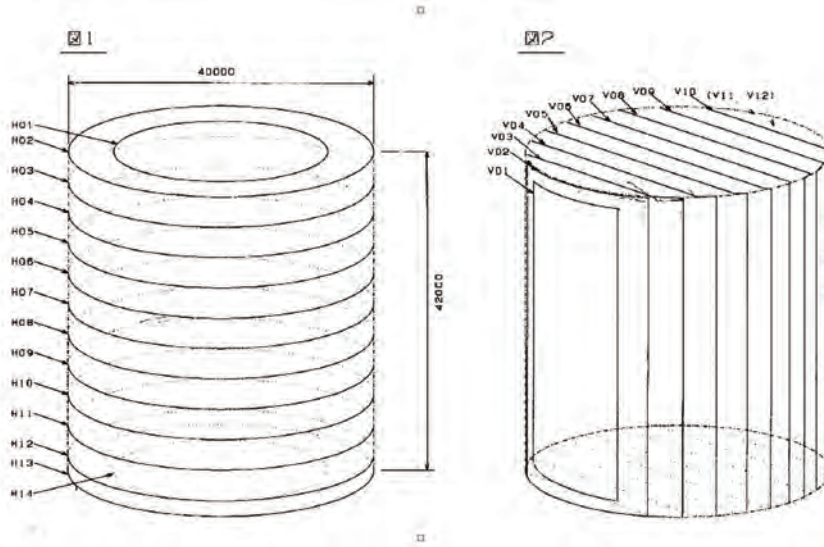


Figure 2.9. Scheme for the horizontal (*left panel*) and vertical (*right panel*) compensation coils installed inside the SK tank together with the naming convention used to label them. The figure is taken from [67].

At two among three locations, part of the coil was successfully bypassed and its functionality was restored. The horizontal location labeled as H05 to H09 was completely underwater, resulting in the entire cable group being turned off. As a consequence of coils failure, for the corresponding data period a 10-20% decrease in collection efficiency was observed for about 20% of PMTs localized in the barrel region of the SK detector. SK prepared the additional works in Summer 2024 in order to install six new horizontal coils. It was a successful operation in which we took part in and were able to restore the geomagnetic field cancellation thanks to all compensation coils working properly. The impact of coils failure on physics analyses is currently under investigation, but we would like to understand it precisely by dedicated calibration steps and fully reliable simulation software, which includes modeling of the field inside the SK tank with and without the presence of the compensation coils.

2.2. The Tokai-2-Kamioka long baseline neutrino oscillation experiment

The Tokai-2-Kamioka (T2K) experiment is a long baseline neutrino oscillation experiment which uses an intense proton beam produced by the Japan Proton Accelerator Research Complex (J-PARC) in Tokai, Japan. It is composed of a neutrino beamline, a near detector complex (ND280) and a far detector SK located 295 km away from J-PARC (see Fig. 2.10).

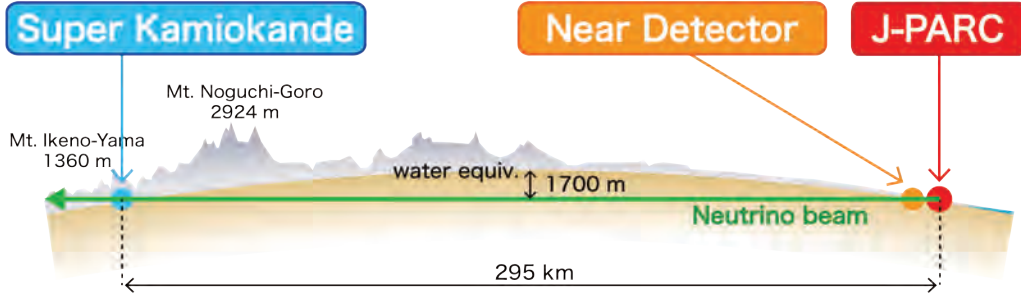


Figure 2.10. Schematic view of the T2K experiment. A neutrino beam is produced at the J-PARC facility and measured by near detectors used to determine the properties of the neutrino beam, and 295 km away far detector SK. Figure is taken from [69]

T2K was designed to observe the *appearance* channel of $\nu_\mu \rightarrow \nu_e$ and the first indication that θ_{13} is not equal to zero, was reported in 2011 with a 2.5σ significance [43]. Two years later T2K has published results reporting that 28 electron neutrino events were detected with an energy distribution consistent with an *appearance* signal, corresponding to a significance of 7.3σ when compared to 4.92 ± 0.55 expected background events [44]. The non-zero θ_{13} mixing angle implies that a measurement of a CP-violating phase, δ_{CP} may be possible in long-baseline accelerator experiments by measuring the appearance of ν_e and $\bar{\nu}_e$ in and ν_μ and $\bar{\nu}_\mu$ beams, respectively. The first results from T2K related to searches of a CP-violating phase, δ_{CP} were published in 2018 in [70]. With an exposure of $14.7(7.6) \times 10^{20}$ protons on target (POT) in neutrino (antineutrino) mode, 89 ν_e candidates and 7 $\bar{\nu}_e$ candidates were observed while 67.5 and 9.0 are expected for $\delta_{CP} = 0$ and normal mass ordering. The obtained 2.0σ confidence interval for the CP violating phase, δ_{CP} , does not include the CP-conserving cases ($\delta_{CP} = 0, \pi$). Few years later, in 2021 we have reported in NATURE [45] article for the first time closed 99.73% (3σ) intervals on the CP-violating phase δ_{CP} (taking into account both mass orderings) and a large range of values around $\delta_{CP} = +\pi/2$ were excluded. Updated results on δ_{CP} searches have also been published in 2023 in [46]. Compared to the previous analysis, an additional 4.7×10^{20} POT neutrino data was collected at the SK. The CP-violating phase is constrained to $\delta_{CP} = -1.97^{+0.97}_{-0.70}$ using Feldman–Cousins corrected intervals, and $\delta_{CP} = 0, \pi$ is excluded at more than 90% confidence level. The most recent results are included in [71] where T2K has made improved measurements of three-flavor neutrino mixing with $19.7(16.3) \times 10^{20}$ POT in (anti-)neutrino-enhanced beam modes. A new sample of muon-neutrino events with tagged pions has been added at the far detector, increasing the neutrino-enhanced muon-neutrino sample size by 42.5%. In addition, new samples have been added at the near detector, and significant improvements have been made to the flux and neutrino interaction modeling.

A comprehensive review of the instrumentation aspect of the T2K experiment may be found in [69]. We will only introduce here its major parts.

T2K is the first long-baseline experiment which adopts an off-axis method [72] to generate the narrow band beam using the new proton synchrotron at J-PARC. In this approach, the neutrino beam is deliberately directed at a given angle with respect to the baseline connecting the proton target and the far detector, SK. The off-axis angle is set at 2.5° . This corresponds to the peak in energy at

0.6 GeV (see Section 2.3.2) which maximizes the effect of the neutrino oscillations at the 295 km as well as minimizes the background to the electron-neutrino *appearance* detection.

The near detector facility, placed at 280 m from the target, consists of on-axis and off-axis detectors. The neutrino beam direction and profile is measured by the on-axis setup (INGRID). Measurements of muon neutrino flux and its energy spectrum as well as intrinsic electron neutrino contamination are the major purpose of the off-axis detectors. All these measurements are vital for the precise characterization of the signal and backgrounds which are measured in the SK. More details of ND280 complex are covered in Section 2.2.1. The far detector of the T2K is SK described in detail in the previous sections. The synchronization between the neutrino event trigger at SK and the beam spill produced at J-PARC is based on the Global Positioning System (GPS). As stated in [69], the accuracy of absolute time determination at two sites is demonstrated to be less than 50 ns. The coincidence of any entering background in Kamioka mine is negligibly low.

On 23rd April, 2009, the first neutrino events from the T2K beam were detected in the SK detector. The first physics run began in January 2010 and lasted till June. After the summer break, continuous running began in November 2010. T2K plans to take physics data until the start of the Hyper-Kamiokande experiment.

2.2.1. J-PARC facility

In the following sections we introduce the J-PARC complex, placed at Tokai, Ibaraki prefecture with its accelerator system, responsible for the production of the T2K neutrino beam, as well as the near detector station with on-axis and off-axis sub-components.

J-PARC accelerator

J-PARC is equipped with three accelerators [72]: a linear accelerator (LINAC), a Rapid Cycle Synchrotron (RCS) and a Proton Synchrotron called Main Ring (MR). The design energies of the three acceleration stages are 400 MeV (181 MeV at present), 3 GeV and 30 GeV, respectively. The number of bunches in MR is eight (six before June 2010). There are two extraction points in the MR: slow extraction for the hadron beamline and fast extraction for the neutrino beamline. The design parameters of the J-PARC MR for the fast extraction are listed in Table 2.3.

Table 2.3. The planned characteristics of MR for the fast extraction in J-PARC for the T2K experiment. Numbers taken from [69]. The J-PARC MR beam power for T2K has increased with recent operation in 2024 achieving over 800 kW.

Circumference	1567 m
Beam power	750 kW
Beam kinetic energy	30 GeV
Beam intensity	3×10^{14} p/spill
Spill cycle	~ 0.5 Hz
Spill width	~ 5 μ sec

Primary and secondary T2K beamline

The neutrino beamline consists of two complementary subsystems as shown in Fig. 2.11. These are the primary and secondary beam lines. In the primary beamline we can distinguish the preparation

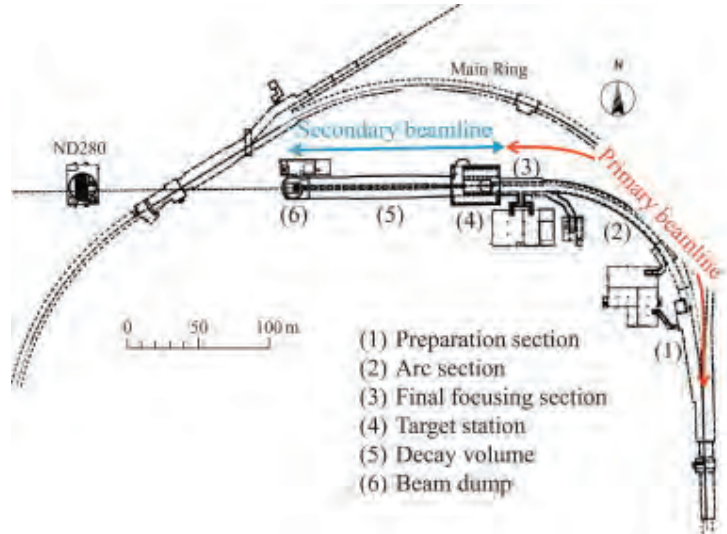


Figure 2.11. T2K beamline at J-PARC. Figure is taken from [69]

section (54 m long), arc section (147 m) and final focusing section (37 m). The preparation section is responsible for tuning the proton beam with the help of 11 normal conducting magnets. The proton beam may later on encounter the arc section where it is bent towards the direction of Kamioka by 80.7° , with a 104 m radius of curvature. In the final focusing section ten superconducting magnets guide and focus the beam onto the target.

A well tuned proton beam is of great importance for the stable neutrino beam production, thus in J-PARC facility there are several beam monitors placed to control the proton beam intensity, profile, position or loss. Details of each monitor may be found in [69].

The author of this monograph was inside the primary beamline of the T2K experiment a few times: in summer 2008 with tests done on the T2K horns, in January 2014 and in September 2016 (see Fig. 2.12).

Three sections constitute the secondary beamline: the target station, decay volume and beam dump (see Fig. 2.13). The target station is equipped with a collimator which protects the three magnetic



Figure 2.12. The author of this monograph was inside the primary and secondary beamlines of the T2K experiment a few times: in summer 2008 with tests done for the T2K horns (secondary beamline), in January 2014 and in September 2016 (primary beamline): (*left panel*) a picture taken by the author with the view of the primary beamline (*right panel*) a picture with the author standing in front of the entrance window to the secondary beamline.

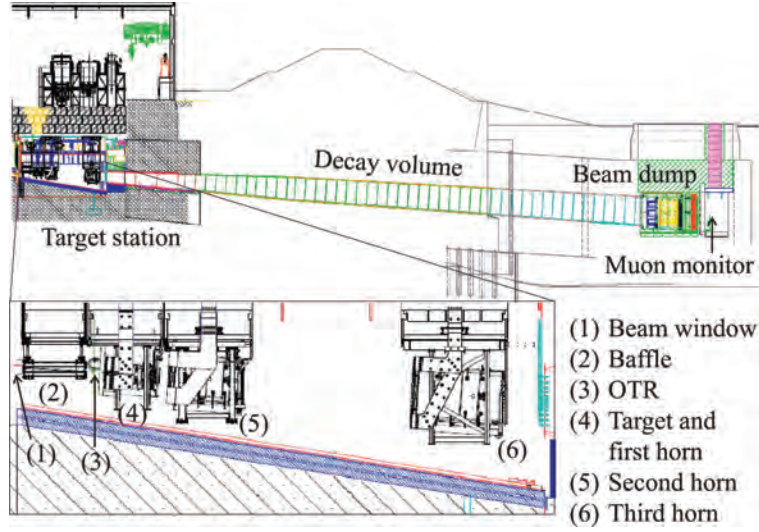


Figure 2.13. Secondary T2K beamline at J-PARC. Figure is taken from [69]

horns, an optical transition radiation monitor (OTR) to control the proton beam profile upstream of the target, the carbon target placed inside the first horn and two additional magnetic horns which together with the first one work currently at 250 kA (designed to handle 320 kA) current pulse to focus secondary charged particles (among which mainly pions and kaons are of interest) produced during $p+C$ interactions.

The T2K target is a 1.9 interaction length (91.4 cm long), 2.6 cm diameter and 1.8 g/cm^2 density graphite rod. It is placed inside the first horn as shown in Fig. 2.14. If the target material were much denser than graphite it would be melted by the pulsed beam head load. When the 0.75 MW proton beam interacts with the target, the temperature at the center is expected to reach 700°C , which is the reason why the target is cooled by helium gas flowing through the gaps between the tube and a 0.3 mm thick titanium case.

The T2K beamline uses three horns to select positively charged particles (mainly pions with admixture of kaons) and bend negative ones. Each magnetic horn is built of inner and outer conductors [73]. The toroidal magnetic field is generated inside the inner conductor and at radius r it is given

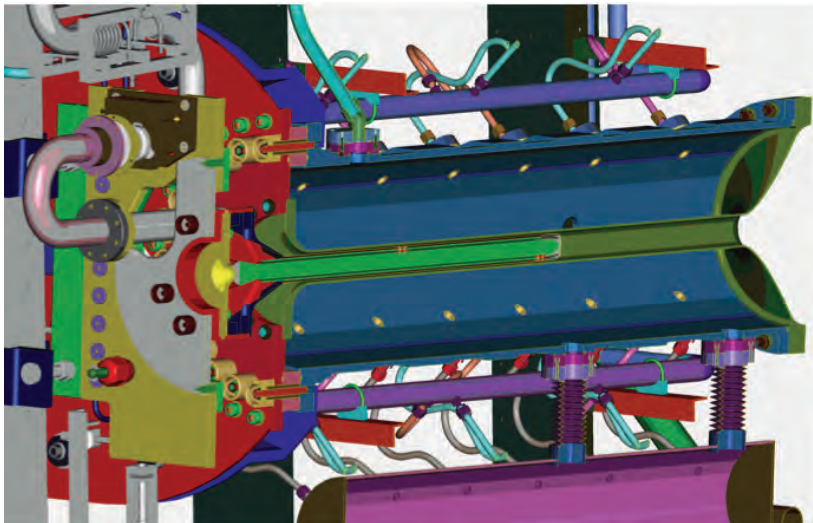


Figure 2.14. Cross section of the first horn with target (green cylinder). Figure is taken from [69]

by the formula:

$$B(r) = \frac{\mu_0 I}{2\pi r} \frac{r^2 - a^2}{b^2 - a^2} \quad (2.1)$$

where μ_0 is the magnetic permeability, I is the current, r is the distance from the horn axis and a and b are, the inner and outer radii of the inner conductor, respectively. The first horn collects secondary positive charged particles, mainly pions, which are produced during p+C interactions. The second and the third horn focuses on secondaries. As stated in [69], when the horns are running with 320 kA currents the neutrino flux at SK at the spectrum peak energy of about 0.6 GeV is increased by a factor of 16 (compared to horns at 0 kA). Thus application of magnetic horns in the T2K experiment is very essential. Note that for $\Delta m_{23}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$ and a baseline of $L = 295 \text{ km}$ the factor $\sin^2(1.27 \Delta m^2 \frac{L}{E})$ reaches its maximum when $1.27 \Delta m^2 \frac{L}{E} = \frac{\pi}{2}$ which corresponds to an energy of $E \simeq 0.6 \text{ GeV}$. This means that the ideal situation would be to study an (almost) monochromatic neutrino beam with an energy of about 0.6 GeV, which—thanks to the ingenuity of physicists and engineers at J-PARC – has in practice been nearly achieved. If the horn current is run in the *forward* direction, positively charged secondaries are focused, which decay to produce a beam that is primarily ν_μ . This is referred to as *forward horn current* (FHC) or neutrino mode. Alternatively the horn current can be reversed, to give a beam of primarily $\bar{\nu}_\mu$, which is referred to as *reversed horn current* (RHC) or antineutrino mode. In either case, secondary hadrons produced in the very forward direction pass through the field-free necks of the horns and contribute to a *wrong-sign* flux that is of order 1% of the intended *right-sign* component in FHC mode and order 10% in RHC mode.

Next on the way is the decay volume, which is 96 m long steel tunnel. Pions decay there into muons and muon neutrinos:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (2.2)$$

Some of the resulting muons can also decay producing muon antineutrinos and electron neutrinos:

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e \quad (2.3)$$

Fewer numerous kaons also decay with the production of electron neutrinos:

$$K^+ \rightarrow \pi^0 + e^+ + \nu_e \quad (2.4)$$

This way an intense neutrino beam is produced and directed towards ND280 and SK. Two last channels are the source of the beam contamination. We would like to have pure muon neutrino beam but clearly we may expect contamination of other neutrinos. At the end of the decay volume we encounter beam dump which absorbs any hadrons as well as muons below 5 GeV/c that reach the end of the tunnel. It is built of graphite blocks cooled with water in aluminum pipes. Particles that pass the dump are, besides the neutrinos of course, high energy muons (those with $p > 5 \text{ GeV}/c$). They are used by the muon monitor situated downstream of the dump to keep an eye on the beam direction.

Near detector complex

As stated earlier, the near detector complex consists of on-axis as well as off-axis subcomponents whose aim is to deliver information on the neutrino energy spectrum, flavor content and interaction rates of the unoscillated beam. At first, let's focus on the on-axis device which is INGRID. It was designed to monitor the neutrino beam direction and intensity directly. INGRID consists of 14 identical modules arranged as a cross of two identical groups along the horizontal and vertical axes. There are also two extra modules located at off-axis directions outside the main cross (see *(left panel)* in Fig. 2.15). These

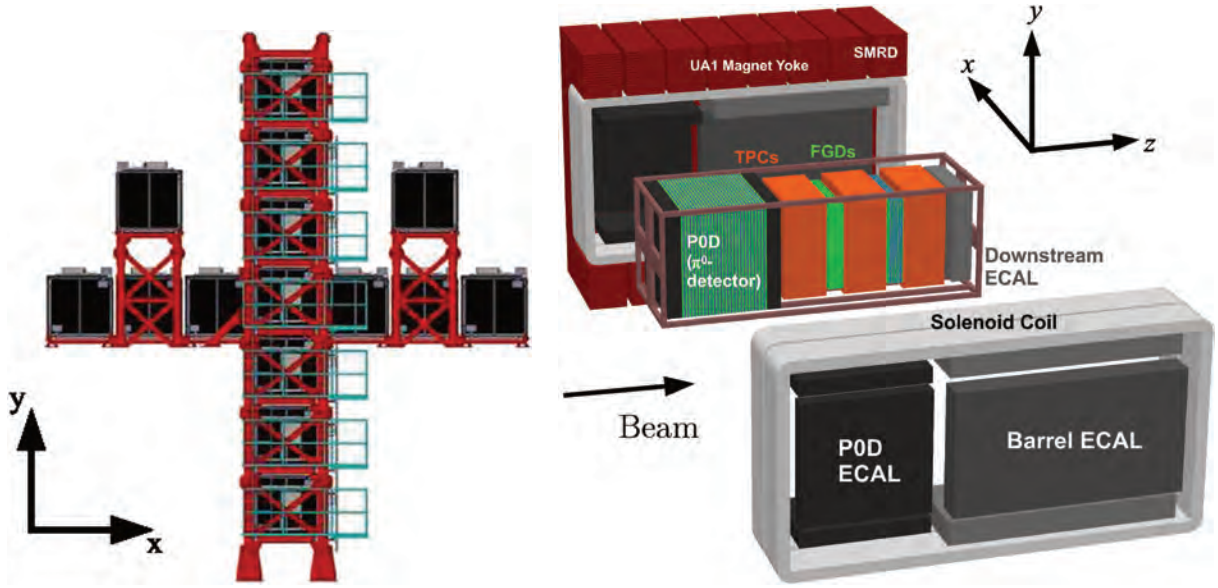


Figure 2.15. The near detector complex with INGRID (*left panel*) and ND280 (*right panel*). Figures are taken from [69]

modules are made of 9 iron plates sandwiched between ten scintillator tracking planes. The center of INGRID cross, with two overlapping modules, corresponding to the neutrino beam center, defined as 0° with respect to the direction of the primary proton beam. INGRID is able to determine the beam direction to better than 1 mrad on a day-by-day basis, which corresponds to a 1 mm shift at the target.

Next to INGRID, the near detector complex is equipped with the off-axis magnetized multipurpose detector ND280. The ND280 off-axis detector must satisfy several requirements. The most important one is to measure ν_μ flux which will be used next in order to determine the ν_μ flux at the SK. Secondly, the ν_e content of the beam must be measured as a function of neutrino energy as it constitutes significant non-removable background in $\nu_\mu \rightarrow \nu_e$ *appearance* search (see Section 2.3.2). Thirdly, it must also measure such ν_μ interactions which may result in additional background to the ν_e *appearance* search at SK. They are dominated by neutral current single π^0 production. All of these requirements were considered while designing the off-axis detector.

The ND280 off-axis detector is composed of several subcomponents, as illustrated in the *right panel* in Fig. 2.15. These are: a water/scintillator detector optimized to identify π^0 from NC interactions of ν_μ (the POD), the tracker consisting of time projection chambers (TPCs) and fine grained detectors (FGDs) optimized to study CC ν_μ and ν_e interactions, and an electromagnetic calorimeter (ECal) that surrounds the POD and the tracker. ECal detects showering particles (like e^- , γ) which are escaping from the inner detectors. All these subcomponents are placed in a 0.2 T magnetic field produced by magnet coming from former experiment UA1/NOMAD at CERN, inside of which we may find also Side Muon Range Detector (SMRD). Plastic scintillation detectors of SMRD measure long range muon trajectories originating from the interactions in the detector. It is also used as a trigger on cosmic ray particles. More detailed description of the ND280 may be found in [69].

2.3. T2K off-axis neutrino beam

In this section, at first we will introduce the advantages of the off-axis neutrino beam over the on-axis configuration. Secondly, predictions of the T2K fluxes at near and far detectors will be discussed using the Monte Carlo simulation tool, jnubeam, which belongs to the T2K software package. The history of the hadron production measurements delivered by NA61/SHINE for the T2K will be presented.

2.3.1. General properties of the off-axis neutrino beams

In order to study the properties of the off-axis neutrino beam we will take into account pion decays which are of major interest for the neutrino beam production. The branching ratio for pions decaying into muons and muon neutrinos (Eq. (2.2)) is 100%. The pion has spin zero and as a result, the decay in its rest frame is isotropic. Neutrino energy in the pion rest frame, denoted as E_ν^* , is constant and equals to:

$$E_\nu^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \simeq 30 \text{ MeV} \quad (2.5)$$

where m_π and m_μ are the charged pion and muon masses, respectively. The laboratory angle between neutrino and pion direction will be denoted as θ . Note that for pions focused by horns in the forward direction this angle will also be a good approximation of the off-axis angle, defined as the angle between primary proton and neutrino beam direction. The θ angle can be expressed by the transverse p_t and longitudinal, p_l , components of the laboratory ν_μ momentum p :

$$\tan \theta = \frac{p_t}{p_l} \quad (2.6)$$

Assuming massless neutrinos, the Lorentz transformation gives:

$$\begin{aligned} p_t &= p_t^* = E_\nu^* \sin \theta^* \\ p_l &= \gamma_\pi(p_l^* + \beta_\pi E_\nu^*) = \gamma_\pi(E_\nu^* \cos \theta^* + \beta_\pi E_\nu^*) = \gamma_\pi E_\nu^* (\cos \theta^* + \beta_\pi) \\ E_\nu &= \gamma_\pi(E_\nu^* + \beta_\pi p_l^*) = \gamma_\pi E_\nu^* (1 + \beta_\pi \cos \theta^*) \end{aligned} \quad (2.7)$$

where all quantities with star denote pion rest frame and others represent laboratory frame. Let's consider relativistic pions $E_\pi \gg m_\pi$ with $\beta_\pi = 1$. Then:

$$p_l = E_\nu = \gamma_\pi E_\nu^* (1 + \cos \theta^*) \quad (2.8)$$

and:

$$\tan \theta \cong \frac{E_\nu^* \sin \theta^*}{E_\nu} \quad (2.9)$$

As $E^* \simeq 30 \text{ MeV}$ is much smaller than the laboratory neutrino energies of interest, we see that the θ angles are very small and limited:

$$\tan \theta < \frac{30 \text{ MeV}}{E_\nu} \quad (2.10)$$

It is also seen that:

$$E_\nu \cong \frac{E_\nu^* \sin \theta^*}{\tan \theta} < \frac{30 \text{ MeV}}{\tan \theta} \quad (2.11)$$

for a fixed θ angle the neutrino energy does not depend on the pion momentum and the maximum E_ν gets smaller with increasing θ angle. On the other hand, one can see that for $\theta = 0$, which implies that $\theta^* = 0$ the neutrino energy is equal to:

$$E_\nu = 2\gamma_\pi E_\nu^* = 2E_\pi \frac{E_\nu^*}{m_\pi} \quad (2.12)$$

neutrino energy E_ν is proportional to the pion energy E_π . In Fig. 2.16 we display effects of these considerations. Note that the small dependence of E_ν on E_π provides a narrow almost mono-energetic distribution of E_ν . In this way one gets a narrow band beam, which is very desirable for precise neutrino experiments like the T2K.

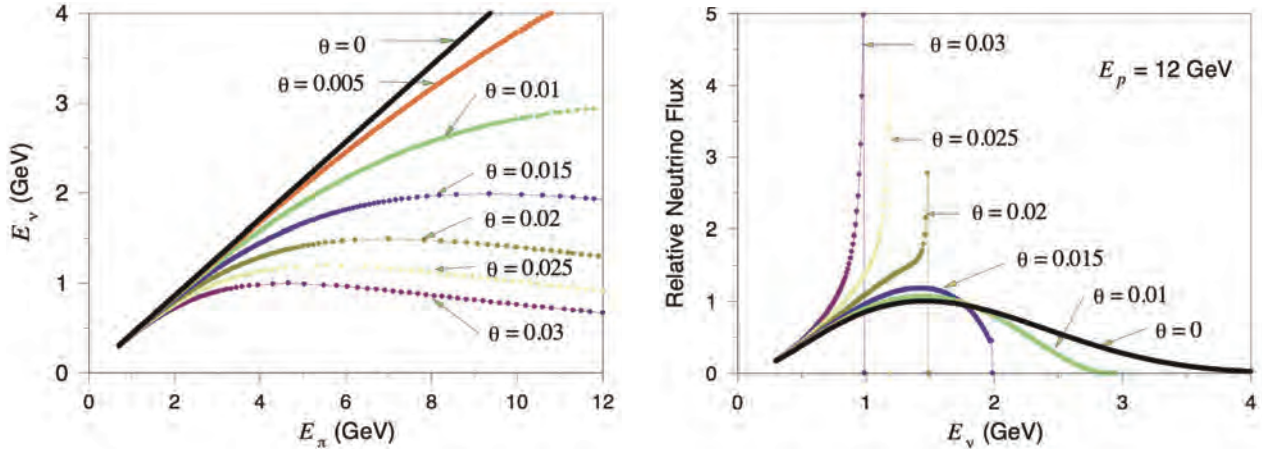


Figure 2.16. Characteristics of the off-axis beam. *Left panel:* neutrino energy as a function of pion energy. *Right panel:* neutrino fluxes versus neutrino energy. Both graphs are performed for a proton with energy 12 GeV interacting with a carbon target. The angle θ is the angle between the pion direction and the produced neutrino direction (in radians). This angle is also a good approximation of the off-axis angle. Figures are taken from [74]

2.3.2. Experience gained in the prediction of the T2K neutrino flux

Neutrino and antineutrino fluxes at the near detector and SK are calculated with data-driven simulation, which takes into account hadron production measurements of 30 GeV protons on a graphite target performed for T2K by NA61/SHINE experiment. In the next subsections we will discuss the history of the hadron production measurements done by NA61/SHINE over the years, how the T2K neutrino flux is predicted, highlighting the places where the author contributed to.

Review of the hadron production measurement for the T2K with NA61/SHINE project

The NA61/SHINE (SHINE = SPS Heavy Ion and Neutrino Experiment) is a large acceptance hadron spectrometer located in the North Area H2 beam line of the European Organization for Nuclear Research (CERN) in Geneva. The experiment uses beams from the Super Proton Synchrotron (SPS). The NA61/SHINE experimental facility is described in detail in [75]. The first measurements performed for the T2K by the NA61/SHINE were carried out using the 2007 pilot run [76] using a graphite target with a thickness of 4% of a nuclear interaction length (so-called thin target) and those data were already used to tune neutrino flux predictions for crucial *appearance* searches in [43, 44] (see Table 2.4). The next publication in 2016 from NA61/SHINE [77] delivered results from the 2009 run on $\pi^\pm$, $K^\pm$, protons, K_S^0 and Λ . The improved statistics of the 2009 run (by a factor of 8 larger with respect to 2007 pilot run) allowed NA61/SHINE not only to improve the precision of the measurements but also to have, for the first time, results of negatively charged kaons. The author of this article was responsible for deriving the double differential cross sections for low momentum $\pi^\pm$ using energy loss measurements in the Time Projection Chambers of the NA61/SHINE detector. The analysis of the data collected during the 2007 pilot run was a subject of the PhD thesis [78]. The second more detailed analysis of $\pi^\pm$ production, was performed for the 2009 data taking period [77]. These data were used up to 2020 in all oscillation results published by T2K [45, 70, 79]. The T2K oscillation analysis, which was using π^+ and π^- yields measured from the replica of the T2K target by the NA61/SHINE [80], was published in 2023 [46].

Methods used to predict the T2K neutrino flux

The method for predicting the T2K neutrino flux and propagating the associated uncertainties is discussed here. We use FLUKA 2011.2x [91, 92] to simulate interactions inside the target. The outgoing

Table 2.4. List of all hadron production measurements taken by NA61/SHINE for the T2K flux predictions. In addition, the citations to the most important T2K publications in which results were using given hadron production measurements from NA61/SHINE are also given.

Year	Target	Spectra	Published by NA61/SHINE(year)	Used by T2K
2007 (pilot run)	thin target	$\pi^\pm$	[76] (2011)	[43, 44, 81] [82–84]
2009	thin target	$\pi^\pm, K^\pm, \Lambda$ K_S^0 , protons	[77] (2016)	[45, 46, 71, 79, 85] [86–89]
2009	T2K replica target	$\pi^\pm$	[80] (2016)	[46, 71]
2010	T2K replica target	$\pi^\pm, K^\pm$	[90] (2019)	[71]

particles from the target, neutrino parents, are tracked through the magnetic horns using the GEANT3-based JNUBEAM package [72]. From the analysis published in [46], the prediction for pions exiting the target’s surface is tuned to π^+ and π^- yields measured by the NA61/SHINE experiment, using data collected in 2009 with a replica of the T2K production target [80]. Pions that leave the target and are within the phase space covered by the replica target data, which is about 90% of the neutrinos at the flux peak are given a weight:

$$w(p, \theta, z, i) = \frac{dn^{\text{NA61}}(p, \theta, z, i)}{dn^{\text{MC}}(p, \theta, z, i)} \quad (2.13)$$

where dn is the POT-normalized differential yield for data (NA61) and simulation (MC) with exciting momentum p , polar angle θ , and longitudinal position z along the target for an exciting particle type $i = (\pi^+, \pi^-)$. For the particles leaving the target, no additional tuning weight is applied for any of the interactions or trajectories inside the target. Simulations for particles that are not covered by the replica target data, and interactions occurring outside the target, are tuned to NA61/SHINE data on $\pi^\pm, K^\pm, K_S^0, \Lambda$ and p yields from a thin target taken in 2009 [77] and other external measurements. In the previous thin target tuning, a large uncertainty on the cross section of proton production was assigned. In the replica target based tuning, this uncertainty is no longer necessary for particles covered by the replica target data, because the exiting particle yields can be tuned directly without knowing the interaction history inside the target. As a consequence, measurements with the T2K replica target can constrain up to 90% of the neutrino flux because hadron yields at the surface of the target are measured not just for the primary interactions but also for the re-interactions inside the target.

The uncertainties from NA61/SHINE are then incorporated with the uncertainties associated with the proton beam profile and out-of-target interactions to give the total uncertainty.

The recent predicted flux distributions are shown in Fig. 2.17. The largest difference from the previous neutrino flux prediction is the reduction of the ν_μ component in the ν -mode and $\bar{\nu}_\mu$ component in the $\bar{\nu}$ -mode (right-sign) by 5–10% for uncertainty on the hadron interactions around the E_ν 0.6 GeV.

In T2K, we are searching for $\nu_\mu \rightarrow \nu_e$ in neutrino beam mode and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ in (anti)neutrino beam mode. The second configuration has a few 3-4% of contamination of ν_μ component for mean T2K neutrino energy 0.6 GeV, which cannot be separated from the original $\bar{\nu}_\mu$ flux. And with the increasing neutrino energy this contamination is also getting higher. This effect can be explained by the fact that we collide protons on a graphite target and already before the interaction, we have more positively charged particles, so after the collision we expect, due to charge conservation, to produce more particles with a positive charge than a negative. Some of those secondary particles will be defocused by the

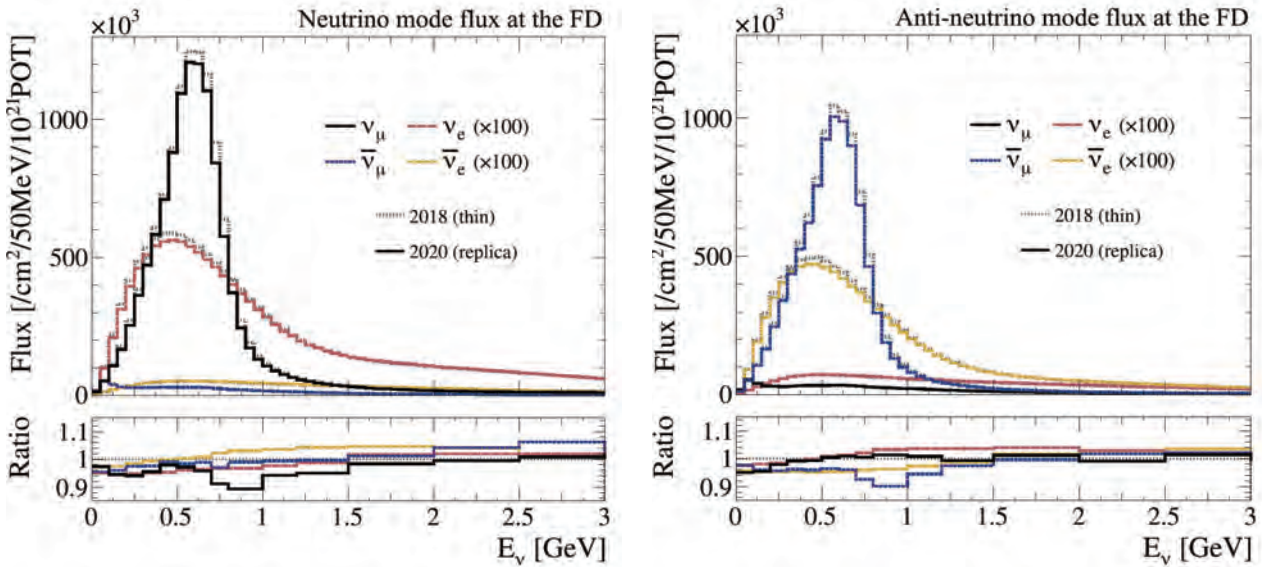


Figure 2.17. The predicted unoscillated neutrino fluxes at the FD of T2K in ν -mode (*left panel*) and $\bar{\nu}$ -mode (*right panel*). The ν_e and $\bar{\nu}_e$ components are scaled by a factor 100. The solid lines show predictions after using NA61/SHINE data on the T2K replica target. The dotted grey lines show predicted flux in the previous analyses, tuned to thin target data of NA61/SHINE. The bottom histograms show the ratio of the flux from the replica target to the flux from the thin target tuning. Figure taken from [46].

magnetic horns placed inside the target station. However, there is a significant fraction of these particles which produced along the beam line will not be defocused from the beam trajectory (in the center of the magnetic horns we have a hole and no magnetic field is present). These particles (π^+ and K^+) will decay and produce ν_μ contamination. As a consequence of this discussion, if ν_μ oscillate to ν_e we can expect to have in SK not only events of interest of originally $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ but also of $\nu_\mu \rightarrow \nu_e$.

To see more clearly wrong-sign and background contributions, the predicted neutrino flux spectra are also shown in logarithmic scale and for a wider range of energies in Fig. 2.18

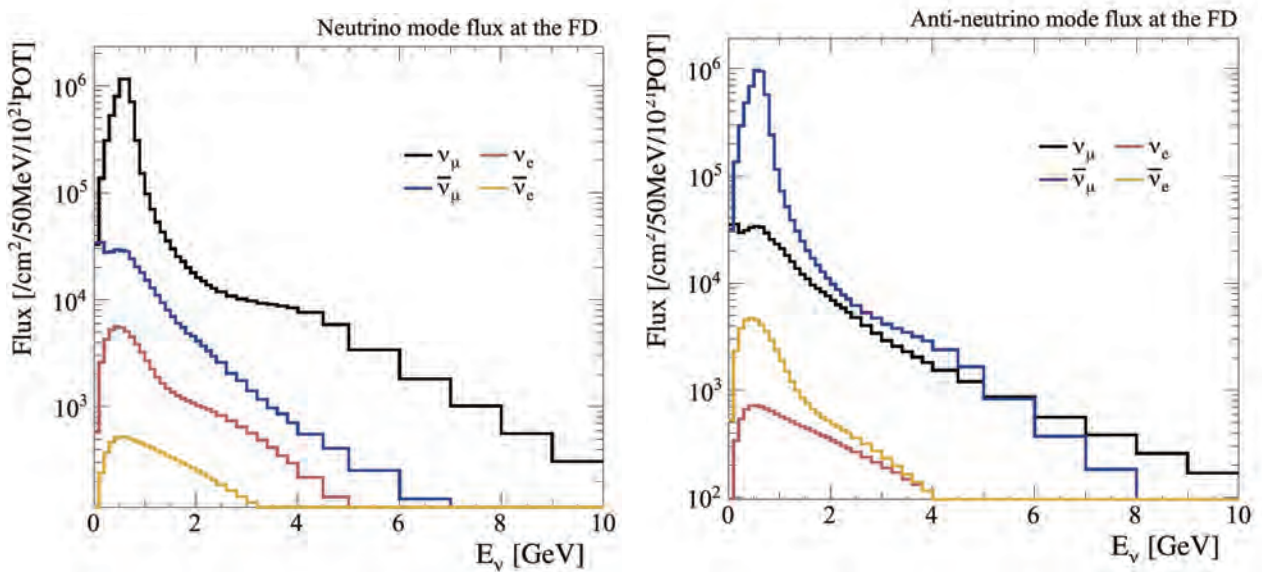


Figure 2.18. The predicted unoscillated neutrino fluxes at the FD of T2K in ν -mode (*left panel*) and $\bar{\nu}$ -mode (*right panel*) in the logarithmic scale with an extended E_ν range, after the tuning to NA61/SHINE data on the T2K replica target. Figure is taken from [46].

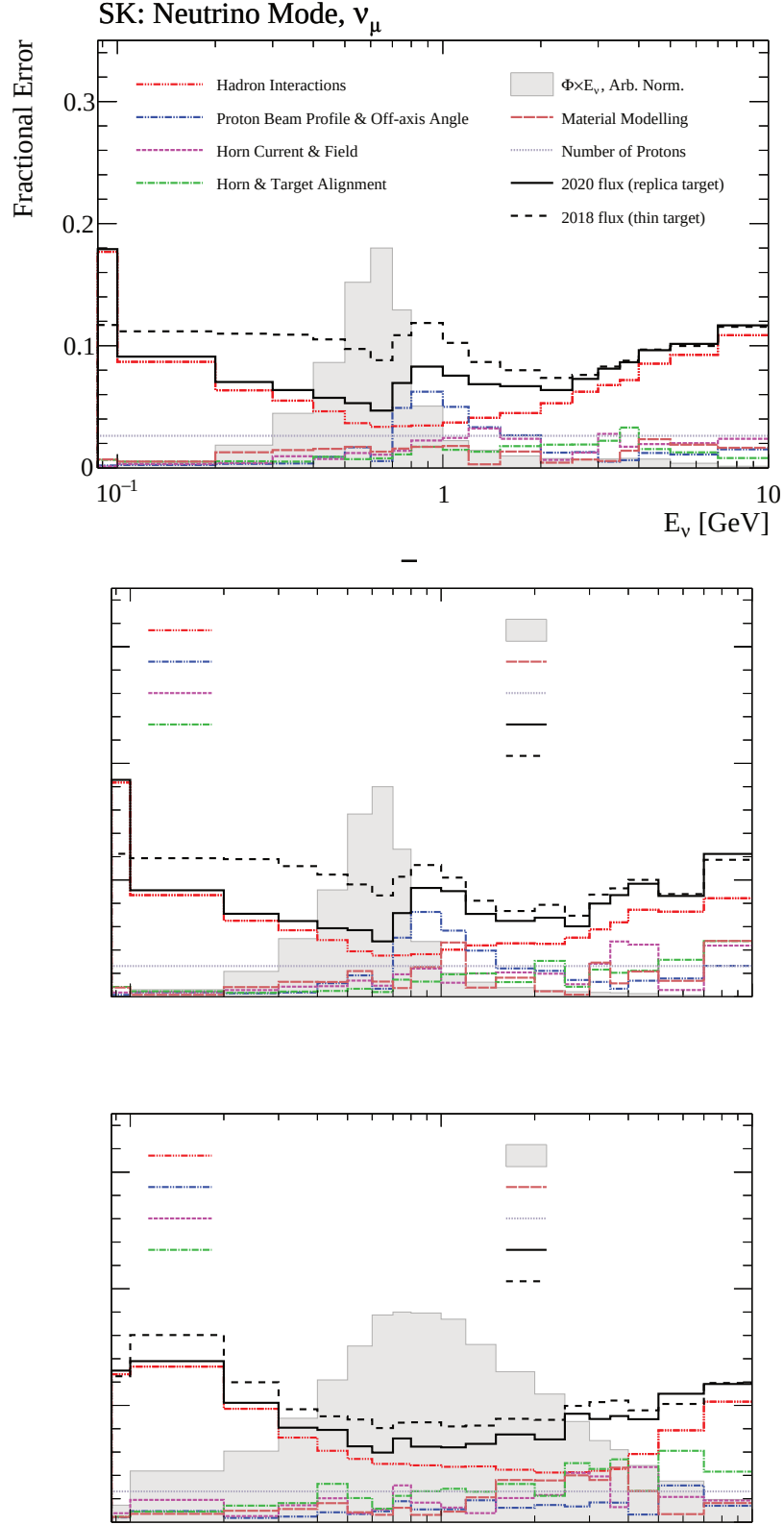


Figure 2.19. Uncertainty on the right-sign flux at SK ν -mode (upper panel) and $\bar{\nu}$ -mode (middle panel) and wrong-sign (bottom panel) fluxes in $\bar{\nu}$ -mode, broken down by the source of uncertainty. The black solid line shows the total T2K flux uncertainty. The dashed black line shows the total T2K flux uncertainty in the previous T2K analyses. Figure is taken from [46].

In the T2K oscillation analysis presented in [46], which will be a subject of the joint oscillation analysis with atmospheric neutrinos from SK (see Chapter 4) overall, tuning with the NA61/SHINE 2009 replica target data using high statistics measurements of $\pi^\pm$ production reduces the uncertainty from 9 to 5% in the flux peak, as seen in Fig 2.19.

At the time when this article was being prepared we had also published the T2K oscillation analysis from 2025 [71]. In that analysis, outgoing $K^\pm$ were also tuned using NA61/SHINE T2K replica target data from 2010 [90]. This reduced the ν_μ flux uncertainty at energies up to 7 GeV below 4%.

In this chapter, we describe a measurement of neutrino oscillation parameters: $\sin^2 \theta_{23}$, Δm_{32}^2 , and δ_{CP} and the unknown neutrino mass ordering using atmospheric neutrino data collected in the Super-Kamiokande (SK) detector over 6511.3 live-days, which corresponds to a total exposure of 484 kiloton-years [93]. This is the first SK atmospheric neutrino oscillation analysis which includes measurements from fully contained (FC) neutrino events from the expanded fiducial volume region, for which the author was working. Total fiducial volume of SK detector is equal to 27.2 kiloton for all SK phases in comparison to 22.5 kiloton for the previous analysis published in 2018 [94]. The total exposure for FC events in this analysis is 484.2 kiloton-years, a 48 % increase over the previous published analysis [93]. In this chapter, we begin with a discussion of the Super-Kamiokande atmospheric neutrino oscillations and flux predictions, continue with the details of the analysis approach, and conclude with the presentation of the main results.

3.1. Atmospheric neutrino oscillations

As introduced in Section 1.3.2 on page 17, atmospheric neutrinos are produced when primary cosmic rays, mainly protons, interact with nuclei in the Earth's atmosphere. Typically, in these high-energy nuclear interactions, numerous $\pi^\pm$ mesons—and, less abundantly, $K^\pm$ mesons—are produced. Since these mesons are unstable, they decay into other particles. For example, a π^+ decays into a muon (μ^+) and a muon neutrino (ν_μ). The produced muon (μ^+) is also unstable and decays into a positron (e^+), a $\bar{\nu}_\mu$ and ν_e . Similar decay processes occur for π^- and $K^\pm$ mesons. The left plot in Fig 3.1 shows the scheme of the production of atmospheric neutrinos.

The atmospheric neutrino energy spectrum extends several orders of magnitude from a few MeV up to TeV. In the GeV energy region, we may expect the ratio of $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e) \simeq 2$. This ratio is predicted to be very close to two by detailed calculations of the neutrino flux. The accuracy of the calculated ratio is estimated to be better than a few percent in the GeV energy range.

The zenith angle, denoted by θ_z , is a key parameter used to describe the path length (baseline) traveled by atmospheric neutrinos before reaching a detector. It is defined as the angle between the direction of the incoming neutrino and the vertical direction (the zenith) at the detector's location. Since atmospheric neutrinos are produced all over the Earth and can easily pass through it, SK observes neutrinos from all directions (see the *left panel* in Fig. 3.1). Neutrinos produced above the detector (downward-going) travel a short distance of about 10 km, while those produced below it (upward-going), on the opposite side of the Earth, travel more than 13,000 km. *Right panel* in Fig. 3.1 shows different

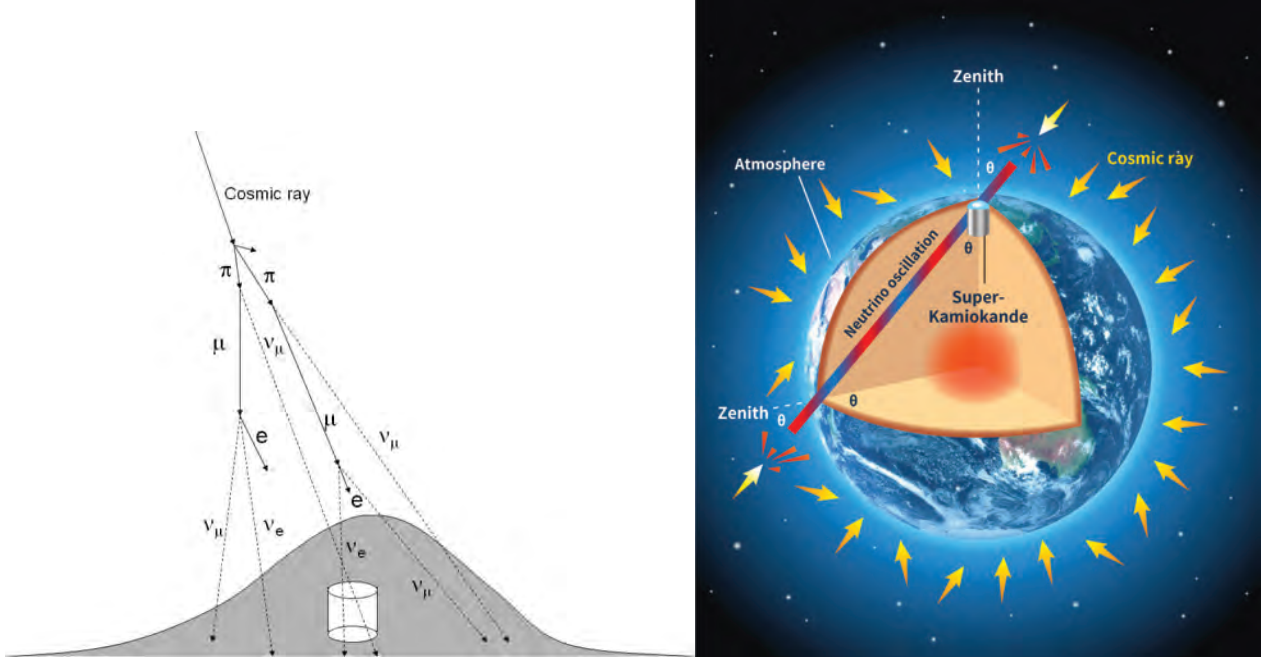


Figure 3.1. *Left panel:* Scheme of the production of atmospheric neutrinos by cosmic-ray interactions with the air nucleus in the atmosphere. Taken from [95]. *Right panel:* Baseline of the atmospheric neutrinos. Taken from [96]

baselines of the atmospheric neutrinos.

In general, the atmospheric neutrino baseline extends from the point where the neutrino is produced in the atmosphere, traverses the Earth, and continues until it reaches the SK detector. Along their path, neutrinos traverse several layers of the Earth with different physical properties (see Table 3.1 for reference).

Table 3.1. Layers that neutrino may flight through during its road to the SK detector. Densities added in the last column are used for the calculation of the neutrino oscillation probabilities, based on a simplified PREM model [97].

Layer	$R_{\min}$ (km)	$R_{\max}$ (km)	Density (g/cm^3)
Atmosphere	6371	–	0
Crust	5701	6371	3.3
Mantle	3480	5701	5.0
Outer core	1220	3480	11.3
Inner core	0	1220	13.0

The analyses presented below use exact three-flavor oscillation probabilities computed with the inclusion of matter effects [98], for varying matter profiles based on a simplified PREM model [97]. The oscillation probabilities inside the Earth are calculated using a constant radial matter density profile, defined as the median density within each of the main regions of the PREM [97] model (see Table 3.1): the inner core ($0 \leq r < 1220$ km), the outer core ($1220 \leq r < 3480$ km), the mantle ($3480 \leq r < 5701$ km), and the crust ($5701 \leq r < 6371$ km). Transition amplitudes are computed across each layer through which the neutrino propagates, and the product of these amplitudes, together with that for traversing the Earth’s atmosphere, is used to obtain the final oscillation probabilities. It was verified that the difference between the probabilities calculated using the simplified model and

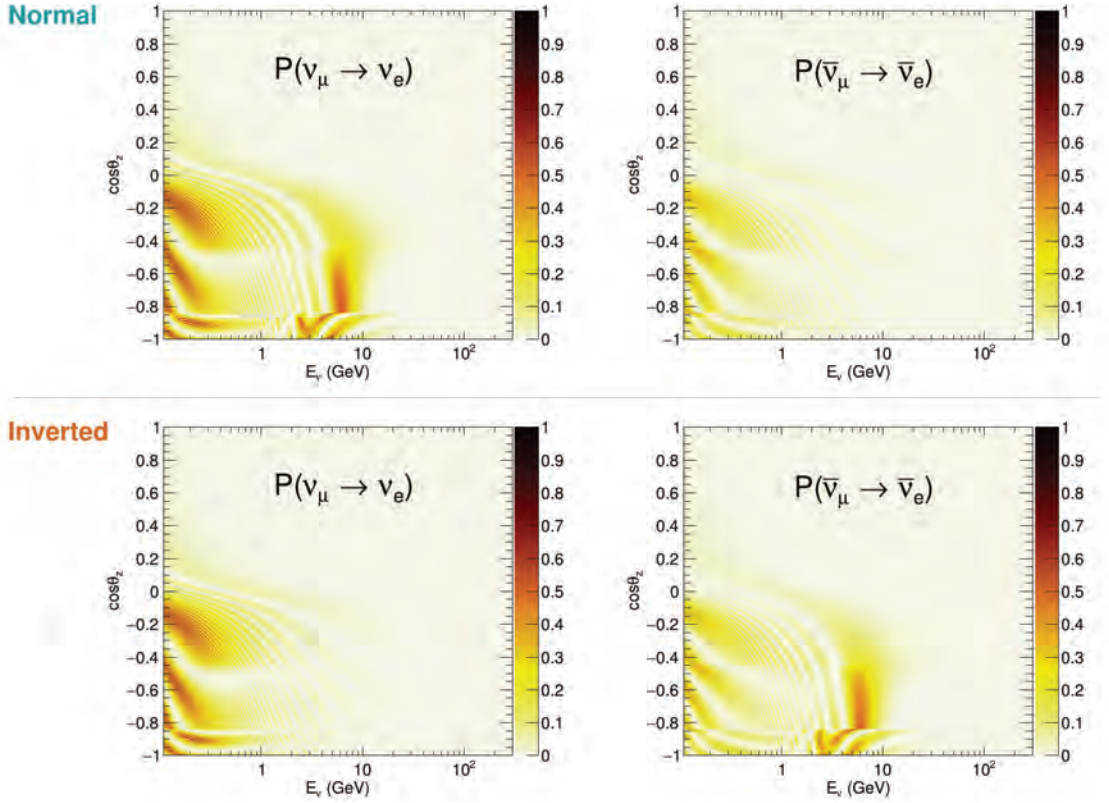


Figure 3.2. Oscillation probabilities of atmospheric neutrinos (*left panel*) and antineutrinos (*right panel*) as a function of cosine zenith angle and neutrino energy. The upper panels illustrate the oscillation probabilities for neutrinos and antineutrinos under the normal mass ordering, whereas the lower panels depict the corresponding probabilities assuming an inverted mass ordering configuration. The probabilities are calculated assuming $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{13} = 0.022$, $\sin^2 \theta_{12} = 0.307$, $|\Delta m_{32,31}^2| = 2.4 \times 10^{-3} \text{ eV}^2$, $\Delta m_{21}^2 = 7.53 \times 10^{-5} \text{ eV}^2$, and $\delta_{\text{CP}} = -\pi/2$. The matter effect resonance is visible in the normal ordering for neutrinos (*upper left panel*) or the inverted ordering for antineutrinos (*lower right panel*) between 2 GeV–10 GeV and for $\cos \theta_z \lesssim -0.5$. Taken from [93].

those obtained with the full PREM model has a negligible impact on the final analysis. Fig. 3.2 shows the calculated oscillation probabilities for atmospheric neutrinos as a function of the cosine of the zenith angle and the neutrino energy E_ν . As discussed in Section 1.2.1 on page 12, Eq. (1.11) shows that the effective quantities depend on the sign of Δm^2 . In particular, for neutrinos in the normal mass ordering ($\Delta m^2 > 0$), $\Gamma \approx \cos 2\theta_{13}$ maximizes the effective mixing angle θ_{13}^M . A maximum also occurs for antineutrinos in the inverted mass ordering ($\Delta m^2 < 0$), as can be seen in Fig. 3.2. This maximum of the effective mixing angle corresponds to a resonant enhancement of muon-to-electron flavor conversions for either neutrinos or antineutrinos, depending on the neutrino mass ordering. Owing to the presence of the matter effects in neutrino oscillations, SK is sensitive to the neutrino mass ordering. If a significant excess of ν_e candidates is observed in the neutrino energy range of 2–10 GeV, accompanied by no visible enhancement in $\bar{\nu}_e$ candidates, this would indicate a preference for the normal mass ordering; the opposite behavior would suggest the inverted ordering. In Fig. 3.2, a discontinuity in the oscillation probabilities can also be seen for cosine zenith angles steeper than -0.9 . This effect corresponds to neutrinos traversing both the outer core and mantle regions of the Earth.

Last, but not least, is the fact that, in order to measure neutrino mass ordering, we need to have the possibility to separate ν_e interactions from $\bar{\nu}_e$ ones in our detector. This is a very challenging task for SK, but not hopeless as described in Section 3.5.1 on page 59.

3.2. Simulation of atmospheric neutrinos

Measurements in SK rely largely on the predictions of expected event distributions. The expectation of atmospheric neutrino events is complex, as it depends on the neutrino flux, interaction cross sections, and the detector response. We use a dedicated Monte Carlo (MC) simulation that accounts for three main steps: (1) calculation of the neutrino flux based on the primary cosmic ray flux and interaction cross sections, (2) simulation of neutrino interactions with nuclei in the detector water, and (3) tracking of secondary particles and simulation of the detector response.

The analysis presented here uses the atmospheric neutrino flux calculated by Honda *et al.* (the Honda flux model [99]) and the neutrino interaction models implemented in the NEUT event generator [100]. The NEUT version 5.6.4 is primarily used; however, for technical reasons, an older version (5.4.0.1) is used for upward-going muons and for the samples from SK-I–III. The tracking of secondary particles and the detector response are simulated using the SKDETSIM program, which has been developed internally by the SK collaboration.

3.2.1. Atmospheric neutrino flux predictions

The SK atmospheric flux model is based on the M. Honda *et al.* calculations described in detail in [99] (referred further as Honda flux). Other models such as G. Battistoni *et al.* [101], (Fluka flux) and G. Bartol *et al.* [102] (Bartol flux) are used as a reference for systematic uncertainties estimation. The primary cosmic ray flux model is determined by experimental measurements, including the BESS [103] and AMS [104] experiments. The cosmic ray flux is affected by solar activities, which change periodically. The flux is higher for an intense solar activity (solar maximum) than that for a mild solar activity (solar minimum). The fluctuation of the cosmic ray flux in a solar cycle is greater than a factor of two in the 1 GeV region, while the difference is around 10 % when energy is around 10 GeV. When primary cosmic ray particles penetrate the atmosphere, they interact with air molecules and produce secondary particles such as pions and kaons. The US Standard Atmosphere ‘76 model [105] is used to describe the profile of the density of the atmosphere and calculate the dependence of atmospheric neutrinos on the zenith angle. The effects of the geomagnetic field are estimated based on the IGRF2005 model [106]. Two theoretical models are used for the hadronic interactions between air molecules and cosmic rays: the JAM [107] and DPMJET-III [108] for energies less than and greater than 32 GeV, respectively. The mesons generated from these hadron interactions decay into muons and neutrinos. Therefore, the DPMJET-III model, which is used in the Honda flux, is tuned based on the measurements of the cosmic-ray muon flux by BESS [103, 109] and L3+C [110].

The atmospheric neutrino MC events are generated using the predicted flux at SK without including oscillations. The effects of neutrino oscillations are then applied by reweighting each MC event according to its corresponding oscillation probability.

The predicted flux is shown in Fig. 3.3. Systematic errors on the atmospheric flux of SK are taken as the difference between the 2011 Honda flux model and the Bartol [102] and Fluka [101] flux models (more details on systematic uncertainties related to atmospheric flux predictions are given in Section 3.6.1 on page 67). This includes errors on the $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$, $\bar{\nu}_e/\nu_e$, and $\bar{\nu}_\mu/\nu_\mu$ ratios, applied as a function of neutrino energy. A normalization error as a function of neutrino energy coming from Honda 2006 [111] is also included.

Furthermore, systematic uncertainties associated with the up/down and horizontal/vertical flux ratios are introduced as functions of the zenith angle, and are treated independently for each SK data sample. An additional energy-dependent uncertainty is applied to the parent meson production ratio, K/π . Further sources of systematic error arise from uncertainties in the neutrino propagation path length, which depend on variations in the atmospheric density profile, as well as from changes in the

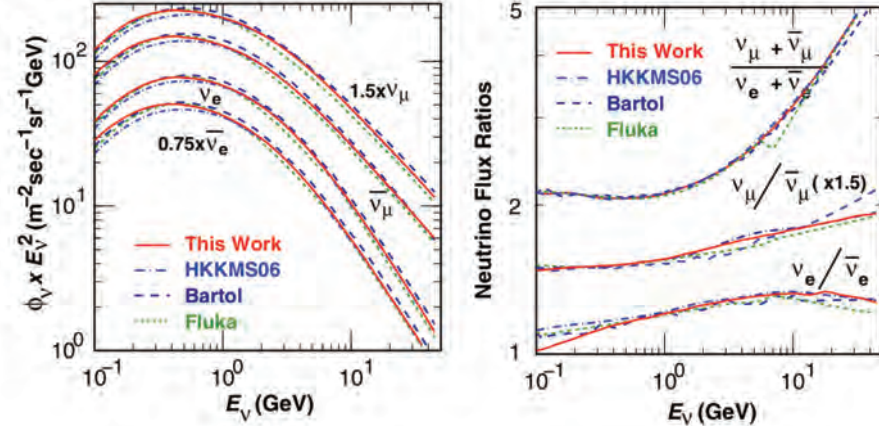


Figure 3.3. Atmospheric neutrino fluxes calculated for Kamioka averaged over all directions (*left panel*), and the flux ratios (*right panel*) from Honda 2011 [99], with comparisons to the Honda 2006, Fluka, and Bartol fluxes. Taken from [99].

neutrino flux due to solar modulation. The Honda flux model provides flux predictions under three solar activity conditions: maximum, minimum, and middle. In the simulation, neutrino events are generated using the flux corresponding to the middle solar condition, and are subsequently reweighted according to the average solar activity during each SK phase. The level of solar activity is estimated from neutron monitor data, which record the flux of cosmic neutrons at ground level [112]. Because the cosmic neutron flux is modulated by the solar wind, it serves as a reliable estimator of the solar activity. Fig. 3.4 presents the temporal evolution of neutron counts and sunspot numbers throughout the entire SK observation period. The relative contributions of solar maximum and minimum conditions for each SK phase are determined by linearly interpolating the average neutron count during that phase between the corresponding values measured at the nearest solar maximum and minimum periods.

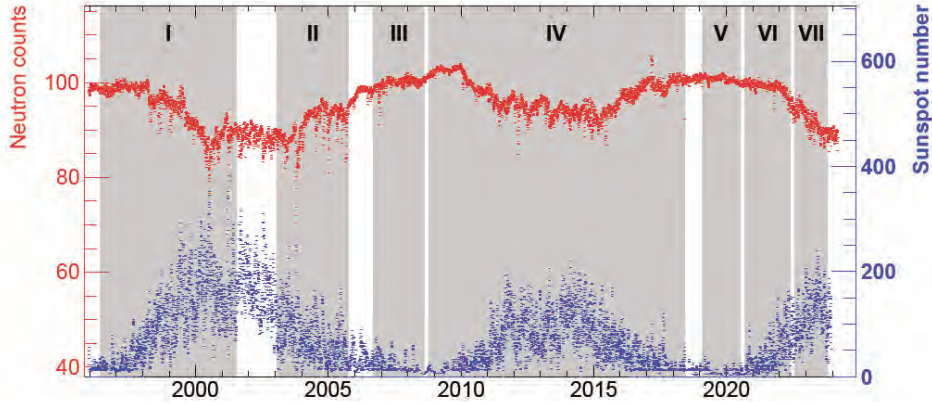


Figure 3.4. Neutron monitor events (red points) [112] and the sunspot events (blue points) during SK observation period. Taken from [113].

3.2.2. Neutrino interactions

At atmospheric neutrino energies, ranging from approximately 100 MeV to $\sim$ TeV, neutrinos interact predominantly with nucleons. For the water target, interactions with free hydrogen nuclei, which are unaffected by nuclear effects, and with bound nucleons in oxygen, which are influenced by effects such as Fermi motion and nuclear binding energy, are simulated separately. Neutrino interactions are classified according to the particles produced in the final state. The main processes relevant to the analysis presented in this work are quasi-elastic (QE) scattering, single-pion production (labeled

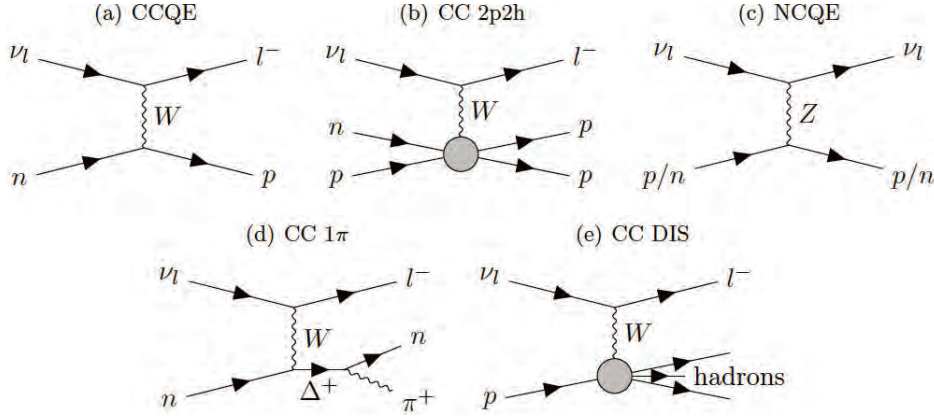


Figure 3.5. Examples of Feynman diagrams for the most important atmospheric neutrino interactions.

as CC1 π or RES), and deep inelastic scattering (DIS). Several important interaction modes, whose corresponding Feynman diagrams are shown in Fig. 3.5, are described in this subsection.

NEUT [100], the official neutrino event generator used in SK, implements several models that predict the interaction cross sections and the kinematics of the outgoing particles for each process. Fig. 3.6 shows the neutrino–nucleon (*left panel*) and antineutrino–nucleon (*right panel*) cross sections as functions of the (anti)neutrino energy. The data are compared with the expectations from the different interaction models described below. The main processes contributing to the total cross section (shown by the black lines) are: quasi-elastic (CCQE, red lines) scattering, resonance (RES, blue lines) production, and deep inelastic scattering (DIS, green lines).

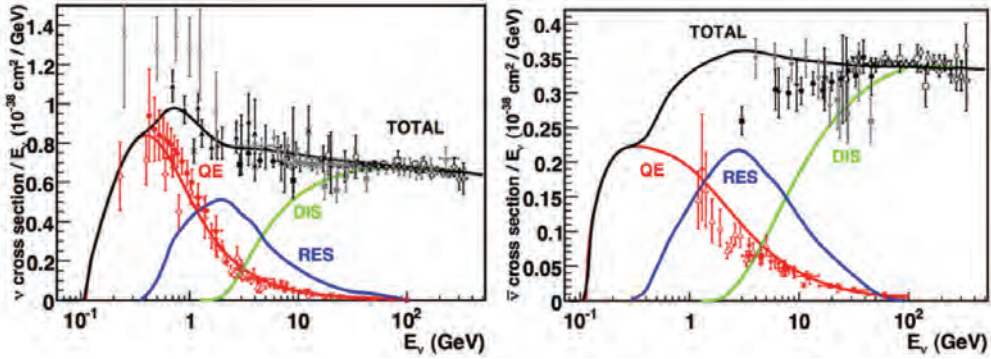


Figure 3.6. The neutrino–nucleon (*left panel*) and antineutrino–nucleon (*right panel*) cross sections plotted as a function of (anti)neutrino energy. The data are compared to the expectations of the different models described in the text. The processes that contribute to the total cross section (shown by the black lines) are: charge-current quasi-elastic (CCQE, red lines) scattering; resonance production (RES, blue lines); and deep inelastic scattering (DIS, green lines). Taken from [114].

Quasi-elastic interactions

The simplest interactions are the two-body scatterings between a neutrino and a nucleon with charge current (CC) or neutral current (NC) exchange:

$$(CC) \quad \nu_l + n \rightarrow l^- + p \quad (3.1)$$

$$(CC) \quad \bar{\nu}_l + n \rightarrow l^+ + p \quad (3.2)$$

$$(NC) \quad \nu_l + n, p \rightarrow \nu_l + n, p \quad (3.3)$$

$$(NC) \quad \bar{\nu}_l + n, p \rightarrow \bar{\nu}_l + n, p \quad (3.4)$$

These interactions are categorized as charge-current quasi-elastic (CCQE) and neutral-current quasi-elastic (NCQE) scatterings. They are referred to as quasi-elastic because the interaction produces only a single outgoing lepton, which carries away most of the momentum relative to the ejected nucleon, while the recoil of the residual nucleus is negligible. The cross section for these processes is formalized in [115] as a function of the four-momentum transfer, q^2 , using the vector and axial form factors of the nucleon. The vector form factors have been precisely measured in electron scattering experiments, whereas the axial form factor can be probed only in neutrino interactions and is therefore more challenging to determine. The axial form factor is typically parameterized in a dipole form as:

$$G_A(q^2) = g_A \left(1 - \frac{q^2}{(M_A^{QE})^2} \right)^{-2} \quad (3.5)$$

where $g_A = G_A(q^2 = 0)$ represents the coupling constant, and that is precisely measured from neutron beta decay. On the other hand, the axial mass M_A^{QE} has a large uncertainty, and the measured values of M_A^{QE} differ as much as 30% [116]. In this analysis, M_A^{QE} is set to 1.21 GeV/ c^2 . In addition to the CCQE scattering, neutrino scattering with a strongly correlated nucleon pair is considered. This process is often called 1p1h or 2p2h as it creates one or two particles and one or two holes, respectively. However, the final state does not necessarily contain only one or two nucleons because of the final state interaction. The 2p2h has been confirmed in electron–nucleus scattering experiments [117], but not yet confirmed directly in the neutrino scattering. In the Valencia model [118] implemented in NEUT, the 2p2h cross section is expected to be a few tens of percent of that of CCQE. As the strong correlation between nucleons is formed by the exchange of mesons, 2p2h is also referred to as a meson exchange current (MEC). Note that 2p2h is considered only for the charged-current interaction in NEUT.

Single meson production

For neutrino energies of approximately 1–several GeV, inelastic scattering processes that produce a single meson become the dominant interaction channels (see Fig. 3.6). Among these, resonant pion production accounts for the majority of the single-meson production cross section, contributing roughly 90% of the total single-pion production rate. In resonant pion production, a pion is emitted from an excited (resonant) state of a nucleon, the lightest of which is the $\Delta(1232)$ resonance. The relevant processes include charge-current (CC) channels such as:

$$\nu_l + p \rightarrow l^- + \Delta^{++} \rightarrow l^- + p + \pi^+, \quad \bar{\nu}_l + p \rightarrow l^+ + \Delta^0 \rightarrow l^+ + p + \pi^-, \quad (3.6)$$

$$\nu_l + n \rightarrow l^- + \Delta^+ \rightarrow l^- + p + \pi^0, \quad \bar{\nu}_l + p \rightarrow l^+ + \Delta^0 \rightarrow l^+ + n + \pi^0, \quad (3.7)$$

$$\nu_l + n \rightarrow l^- + \Delta^+ \rightarrow l^- + n + \pi^+, \quad \bar{\nu}_l + n \rightarrow l^+ + \Delta^- \rightarrow l^+ + n + \pi^-, \quad (3.8)$$

and NC channels:

$$\nu_l(\bar{\nu}_l) + p \rightarrow \nu_l(\bar{\nu}_l) + \Delta^+ \rightarrow \nu_l(\bar{\nu}_l) + p + \pi^0, \quad (3.9)$$

$$\nu_l(\bar{\nu}_l) + p \rightarrow \nu_l(\bar{\nu}_l) + \Delta^+ \rightarrow \nu_l(\bar{\nu}_l) + n + \pi^+, \quad (3.10)$$

$$\nu_l(\bar{\nu}_l) + n \rightarrow \nu_l(\bar{\nu}_l) + \Delta^0 \rightarrow \nu_l(\bar{\nu}_l) + p + \pi^-, \quad (3.11)$$

$$\nu_l(\bar{\nu}_l) + n \rightarrow \nu_l(\bar{\nu}_l) + \Delta^0 \rightarrow \nu_l(\bar{\nu}_l) + n + \pi^0 \quad (3.12)$$

Here, Δ can be other excited barion. To calculate the resonant pion production cross section, one needs to add the contributions from each baryon resonance that produces the desired final state. The largest contribution to these resonances at few-GeV energies is the $\Delta(1232)$, which is the lowest-mass resonance above the pion production threshold. However, there are 18 such resonances below 2 GeV.

NEUT uses the Rein-Sehgal model [119] for resonant pion production:

$$\frac{d^2\sigma}{dq^2 dW} = \frac{1}{32\pi m_N E_\nu^2} \frac{1}{2} \sum_{\text{spin}, j} |T(\nu N \rightarrow l N_j^*)|^2 \frac{1}{2\pi} \frac{\Gamma_j}{2\pi(W - M_j)^2 + \Gamma_j^2/4} \quad (3.13)$$

Here, q^2 denotes the four-momentum transfer, W is the invariant mass of the resonant state, m_N is the mass of the target nucleon, and E_ν represents the neutrino energy. The parameters M_j and Γ_j correspond to the mass and decay width of the excited baryon N_j^* , respectively. The factor $T(\nu N \rightarrow l N_j^*)$ describes the amplitude for producing the resonant state, while the final term is the Breit–Wigner factor representing its decay. The summation runs over the possible spin states of N_j^* and includes 18 resonances with $W < 2$ GeV (natural units: $c = 1, \hbar = 1$). The form factors used in the model are based on the Graczyk–Sobczyk parametrization [120, 121]. Similar to the quasi-elastic case, the resonant pion production cross section also contains an axial form factor:

$$C_A^5(q^2) = C_A^5(0) \left(1 - \frac{q^2}{(M_A^{\text{RES}})^2}\right)^{-2} \quad (3.14)$$

where both parameters: the resonant pion production axial mass, M_A^{RES} , and the coefficient, $C_A^5(0)$, are tuned with the bubble chamber data of $\nu_\mu + p \rightarrow \mu^- + p + \pi^+$ [122] to be 0.95 GeV/ c^2 and 1.01, respectively. In NEUT 5.4.0, value of the axial mass for resonant single pion production, is set to $M_A^{\text{RES}} = 0.95$ GeV/ c^2 , the form factor coefficient is set to $C_A^5(0) = 1.01$.

Coherent pion production. Another process to produce a single pion is a coherent pion production, where neutrinos interact with the whole nucleus instead of a nucleon:

$$(\text{CC}) \quad \nu_l + A \rightarrow l^- + \pi^+ + A \quad (3.15)$$

$$(\text{CC}) \quad \bar{\nu}_l + A \rightarrow l^+ + \pi^- + A \quad (3.16)$$

$$(\text{NC}) \quad \nu_l + A \rightarrow \nu_l + \pi^0 + A \quad (3.17)$$

$$(\text{NC}) \quad \bar{\nu}_l + A \rightarrow \bar{\nu}_l + \pi^0 + A \quad (3.18)$$

Here, A denotes the target nucleus. This process occurs at low q^2 , where only a small amount of momentum is transferred to the nucleus, resulting in the outgoing lepton and pion being scattered predominantly in the forward direction. The Berger–Sehgal model [123] is implemented in NEUT to simulate coherent pion production. The predicted cross section for this process is significantly smaller than that of resonant pion production.

Deep inelastic interactions

Deep inelastic scattering (DIS) becomes the dominant interaction process for neutrinos with energies $E_\nu \geq 5$ GeV. DIS interactions are characterized by a neutrino interacting directly with a quark, thereby breaking apart the nucleon and producing multiple hadrons in the final state. In NEUT, any interaction that results in two or more hadrons in the final state is classified as DIS. For $W < 2$ GeV, the charged-current (CC) DIS cross section is calculated using the formulation of Albright and Jarlskog [124]:

$$\frac{d^2\sigma}{dx dy} = \frac{G_F^2 M E}{\pi} (B F_2(x) \pm C x y F_3(x)), \quad \text{where} \quad (3.19)$$

$$B = 1 - y + \frac{1}{2}y^2 + \frac{m^2}{4ME} \frac{y-2}{x} - \frac{M}{2E}xy - \frac{m^2}{4E^2} \quad (3.20)$$

$$C = 1 - \frac{1}{2}y - \frac{m^2}{4ME_x} \quad (3.21)$$

Here, E denotes the energy of the incoming neutrino, and M and m represent the masses of the target nucleon and the outgoing charged lepton, respectively. The negative (positive) sign corresponds to neutrinos (antineutrinos), reflecting the fact that neutrino and antineutrino interactions produce different amounts of hadronic energy for the same incident neutrino energy. The variables $x = \frac{-q^2}{2M(E-E_l)}$ and $y = \frac{E-E_l}{E}$ are the Bjorken scaling variables, representing the elasticity and the fractional energy loss in the interaction, respectively. The parton distribution functions (PDFs) F_2 and F_3 are taken from [125], with Bodek–Yang corrections [126] applied in the low- q^2 region. For $W > 2$ GeV, PYTHIA 5.7 [127] is used to model the hadronization process, employing the same PDFs as described above.

Final state interactions

The outgoing particles, particularly hadrons produced in neutrino interactions, can undergo modifications due to re-scattering, both inside and outside the target nucleus. Interactions occurring within the target nucleus are referred to as final state interactions (FSIs) and are accounted for in NEUT. Interactions with other nuclei outside the target nucleus are known as secondary interactions (SIs) and are included in the detector simulations.

In NEUT, the following four hadronic processes are modeled as FSIs:

- **Scattering:** Hadrons are elastically scattered by surrounding nucleons, resulting in changes to their momentum and direction.
- **Charge exchange:** Hadrons exchange charge through scattering with other nucleons. For example, $\pi^+ + n \rightarrow \pi^0 + p$.
- **Absorption:** Hadrons are absorbed within the target nucleus.
- **Production:** Additional hadrons, particularly pions, are produced through inelastic scattering processes.

The normalization of each channel is determined by fitting the model to various hadron–nucleus scattering data [128].

The FSIs are simulated using a semi-classical intranuclear cascade (INC) model, in which each hadron is tracked step by step as it propagates through the nucleus until it escapes [129]. At each step, the treatment of the particle is determined according to the mean free paths corresponding to the interaction channels described above. In these calculations, the nucleus is modeled as a local Fermi gas with a Woods–Saxon nucleon density distribution.

3.2.3. Detector simulation

Neutrino interactions simulated by NEUT ultimately produce lists of final-state particles and their corresponding momenta. To enable direct comparison with SK data, these particles must be further propagated through the SK detector. This is achieved using the SKDETSIM program, which is based on the GEANT3 software package [130]. SKDETSIM simulates secondary interactions and particle transport in the detector water, the production and propagation of Cherenkov photons, and the detector response, including photomultiplier tubes (PMTs) and readout electronics. The water properties and detector response parameters are tuned using calibration data obtained from the detector.

Secondary interactions

Secondary interactions (SIs) occurring outside the target nuclei are simulated in the detector simulation in a manner similar to FSIs in NEUT. For low-momentum charged pions with momenta below 500 MeV/c, NEUT is called directly. Hadronic interactions in other cases are simulated using the

CALOR package [131], which employs HETC (High-Energy Transport Code) [132] for hadrons below 10 GeV, FLUKA [101] for hadrons above 10 GeV, and MICAP (Monte Carlo Ionization Chamber Analysis Package) [133] for neutrons below 20 MeV.

3.3. Neutrino event reconstruction

The neutrino event reconstruction algorithm used in this analysis, APFIT, was developed at the beginning of the SK experiment. APFIT played a crucial role in the discovery of atmospheric neutrino oscillations in 1998 [31] and has also been employed in both the K2K [39] and T2K [44] experiments. APFIT is a single-iteration fitter that utilizes the timing and charge information recorded by the photomultiplier tubes (PMTs) [66].

The interaction vertex is reconstructed from the PMT hit timing information after applying a time-of-flight correction for photons. The direction of the first identified Cherenkov ring—typically the one with the highest energy—is then determined from the observed charge distribution relative to the reconstructed vertex. Additional ring candidates are found using a Hough-transform-based method and are selected with an optimized likelihood function designed to reject spurious rings.

Once the number of Cherenkov rings in the event is established, the particle type associated with each ring is identified based on the ring pattern and opening angle. Muons and electrons are distinguished by their characteristic ring morphologies: muons generally produce *sharper* rings due to minimal scattering, while electrons form *fuzzier* rings as a result of their electromagnetic showers.

In the final stage of the fit, the momentum of each Cherenkov ring is estimated from the charge information recorded by the PMTs whose angular positions relative to the line connecting the interaction vertex and the ring center are less than 70° . Corrections and adjustments are applied when two rings overlap. It is worth noting that in APFIT, the hit timing information is utilized only in the initial step to determine the vertex candidate and is not used in subsequent steps such as particle identification or ring counting.

3.4. Expansion of the fiducial volume

We begin by introducing the so-called fully contained (FC) neutrino events, defined as those whose interaction vertex is reconstructed within the inner detector (ID) of SK and which exhibit no clusters of hits in the outer detector (OD) region. The fiducial volume (FV) of the SK detector is defined using the distance between the reconstructed vertex and the nearest ID wall (*dwall*), independent of the reconstructed particle direction. In the previous analysis [94], this region was defined for FC events as being more than 200 cm from the ID wall ($200 \text{ cm} < d_{\text{wall}}$), corresponding to a fiducial mass of 22.5 kilotons—commonly referred to as the conventional fiducial volume.

In this analysis, FC neutrino events reconstructed within the additional 4.7 kiloton region, corresponding to a distance range of 100–200 cm from the ID walls, are also included for all SK phases. To incorporate events within this extended fiducial volume, both the reconstruction algorithms and background estimations were re-evaluated, following procedures first developed for the proton decay searches reported in [134].

To expand the FV, it is necessary to study the impact of background events on the selected analyses within the enlarged region of $100 \text{ cm} < d_{\text{wall}} < 200 \text{ cm}$. The most critical sources of background in this region are cosmic-ray muons and noise events originating from discharges in the PMTs dynodes, commonly referred to as *flasher* events. Both of these belong to the category of non-neutrino background events. As reported in [134], all FC events with reconstructed vertices located more than 50 cm from

the ID wall were visually inspected to estimate contamination from non-neutrino backgrounds. Due to the finite vertex resolution, some cosmic-ray muons and *flasher* events can be reconstructed at apparent distances greater than 50 cm from the ID wall. Naturally, their occurrence becomes more frequent as the true vertex position approaches the SK walls.

Cosmic-ray muons are present in the sample because they can pass through the cable bundles running across the OD without producing sufficient light to trigger the PMTs before entering the ID. To account for this, plastic scintillator veto counters are installed above the four cable bundles. Events depositing a large charge in these veto counters are removed from the sample. Additionally, events whose reconstructed direction is downward-going and whose vertex is located within 2.5 m of any of the twelve ports containing cable bundles, are also rejected. As a result, the fraction of penetrating cosmic-ray muons is significantly reduced. Michel electrons originating from the decay of sub-Cherenkov-threshold cosmic-ray muons that are invisible in the ID are also rejected. This is accomplished by removing all events that have more than 50 OD hits within a 200 ns time window prior to the ID trigger. The remaining non-neutrino background is shown in Fig. 3.7.

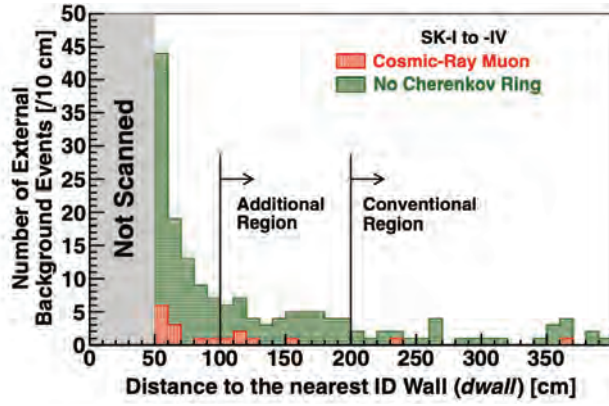


Figure 3.7. Distribution of the distance to the nearest ID wall from the reconstructed vertex (d_{wall}) of non-neutrino background events within the fiducial volume. The figure includes all FC events with reconstructed vertices more than 50 cm from the wall for which eye-scanning was performed. Cosmic ray muon events are marked with a red histogram and events without Cherenkov rings are green (so called PMT *flasher* events). Taken from [134]

As it was estimated in [134], the fraction of non-neutrino background relative to atmospheric neutrino events is equal to 0.5% in the additional FV compared to 0.1% in the conventional FV region. Within the range $100 \text{ cm} < d_{wall} < 200 \text{ cm}$, this fraction increases to about 2.0%, which limits the possibility of further expanding the fiducial volume with the APFIT reconstruction software.

As the reconstructed interaction vertex approaches the ID walls, the number of PMTs affected by the Cherenkov cone decreases, and the hit PMTs become more closely clustered compared to interactions occurring at least 200 cm from the wall. When PMTs hits are too localized, the reconstruction algorithm may struggle to resolve separate Cherenkov rings, which affects the classification of events into single- and multi-ring categories. Furthermore, particles exiting the ID may have biased reconstructed momenta, and tagging Michel electrons from their decays becomes nearly impossible. These effects collectively influence the particle identification performance in the SK detector.

For events with fewer ID PMTs hits, the calculation of the expected charge for each PMTs becomes especially important. To account for these effects, the expected charge tables have been updated using events both inside and outside the conventional FV, parameterized as functions of (1) the distance between the event vertex and each PMTs, and (2) the angle between the particle direction and the PMTs viewing axis. The use of these updated charge tables reduces the $e-\mu$ misidentification rate for single-ring events in the additional FV region by up to 35 %.

To evaluate the detector performance, energy scale measurements are performed for each SK period using various calibration samples that cover a wide range of energies in both fiducial volume (FV) regions. Neutral pions reconstructed from atmospheric neutrino interactions provide a calibration point for the π^0 mass. In addition, stopping cosmic-ray muons with different momenta is used to measure the photoelectron yield as a function of the muon track length (or Cherenkov angle) across both the Sub-GeV and Multi-GeV energy ranges. The muon track length is determined from the reconstructed primary vertex and the position of the Michel electron produced in the muon decay. Finally, Michel electrons themselves serve as a low-energy calibration sample.

In the oscillation analysis presented in this chapter, the absolute energy scale uncertainty, shown in Fig. 3.8, is conservatively taken to be the largest measured value for each SK period.

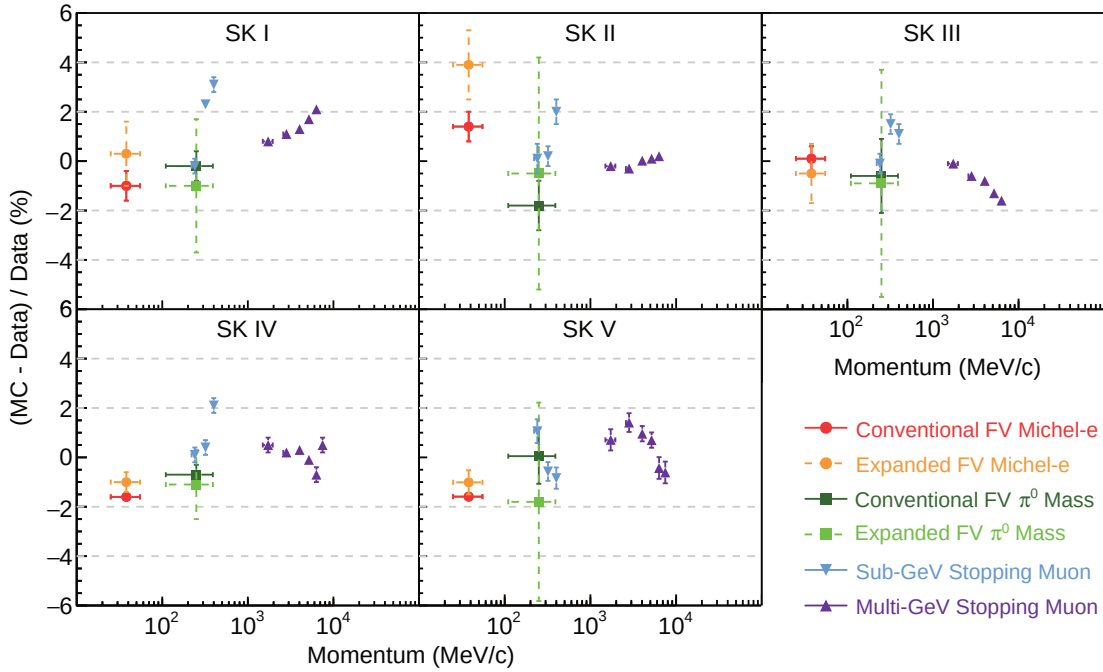


Figure 3.8. Absolute energy scale measurements for each SK period and two FV regions. Vertical error bars indicate the statistical uncertainty and horizontal error bars the momentum range covered by each sample. Taken from [93]

It should be noted that [134] included events from the additional fiducial volume region in the context of a proton decay search, which primarily focused on Sub-GeV atmospheric neutrinos backgrounds. In contrast, the oscillation analysis presented here incorporates both Sub-GeV and Multi-GeV atmospheric neutrino events.

Additional studies beyond those reported in [134] were therefore required to validate the performance of the reconstruction algorithms for Multi-GeV events in the extended fiducial volume region. The reconstructed momentum and direction biases and resolutions were examined using Monte Carlo (MC) samples, confirming that these quantities are consistent between events in the conventional and additional fiducial volume regions.

Furthermore, the author of this work studied the agreement between data and MC for both the ring particle identification (PID) and ring-counting likelihoods in the additional fiducial volume region, as illustrated in Fig. 3.9, along with the full set of data–MC comparisons for all fully contained (FC) neutrino samples used in SK, which are discussed in the following sections.

Fig. 3.9 compares the data and MC distributions for Sub-GeV events (defined by the visible energy in the SK detector as $E_{\text{vis}} < 1330.0$ MeV, see Section 3.5 on page 57) and Multi-GeV events ($E_{\text{vis}} \geq 1330.0$ MeV) in both fiducial volume regions: conventional and expanded. The level of agreement

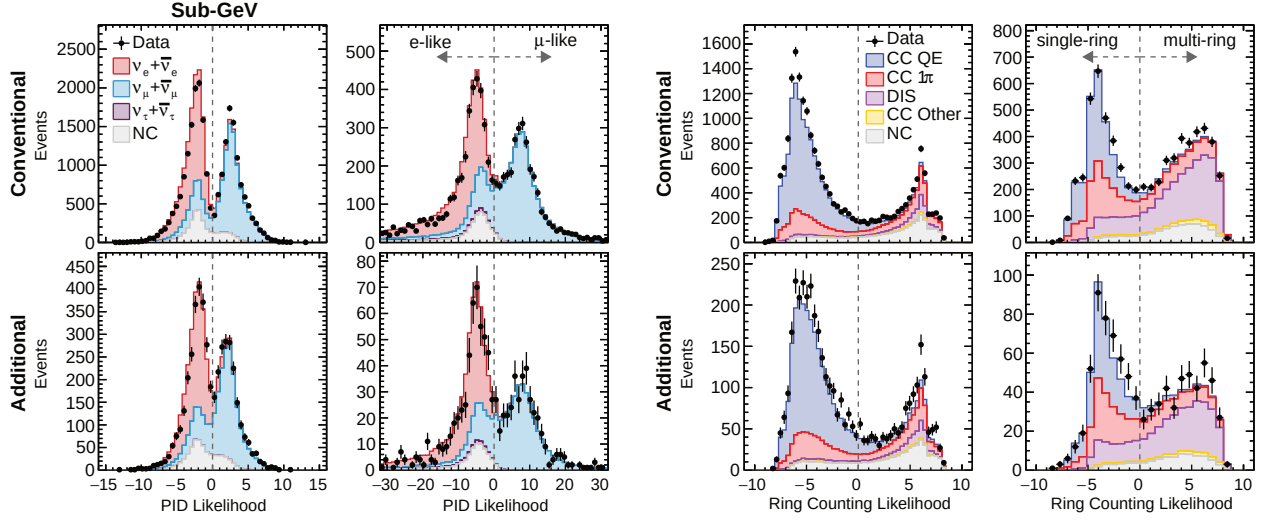


Figure 3.9. SK-IV-V data and MC distributions of the likelihood variables used to separate events into distinct analysis samples. The *upper panels* correspond to events contained within the standard fiducial volume, while the *lower panels* show events reconstructed in the extended fiducial volume. The *first and third columns* display Sub-GeV samples (defined by a visible energy of $E_{\text{vis}} < 1330$ MeV), whereas the *second and fourth columns* correspond to Multi-GeV samples ($E_{\text{vis}} \geq 1330$ MeV). The *left four panels* present the particle identification (PID) likelihood for the most energetic Cherenkov ring in each event: negative values indicate e-like rings, while positive values correspond to μ -like rings. The *right four panels* show the ring-counting likelihood, where negative values denote single-ring topologies and positive values correspond to multi-ring topologies. In these plots, the CCQE category includes both 1p1h and 2p2h interaction modes, while the “CC Other” classification refers to single non-pion hadron production. All MC spectra are shown with neutrino oscillations applied, and no systematic uncertainties are included. Figure taken from [93]

in the likelihood distributions obtained with the updated reconstruction algorithms is found to be satisfactory across both fiducial volume regions and for both Sub-GeV and Multi-GeV energy ranges.

3.5. Atmospheric neutrino sample selections

Neutrino events observed in SK are categorized as fully contained (FC), partially contained (PC), or upward-going muons (Up- μ). The final event rates for each category as a function of time since the start of SK operations are shown in Fig. 3.10. On average, approximately 10 FC, 1 Up- μ , and 0.5 PC events are recorded per day in the SK detector.

True atmospheric neutrino energy spectra from simulation used in the SK detector for all three categories of the events: fully-contained (FC), partially-contained (PC) or upward-going muons (Up- μ) are shown in Fig. 3.11.

FC events are defined as those with reconstructed vertices located within the 27.2 kton expanded fiducial volume, corresponding to the region more than 1 m from the inner detector (ID) wall (see Section 3.4 on page 54), and with no activity observed in the outer detector (OD). The FC data are further divided into Sub-GeV and Multi-GeV samples according to the visible energy E_{vis} . Events with $E_{\text{vis}} < 1330.0$ MeV are categorized as Sub-GeV.

The number of reconstructed Cherenkov rings is then used to further classify events into single-ring and multi-ring types. For single-ring events, the particle identification (PID) of the most energetic ring is used to separate e-like and μ -like samples. Additional selection criteria for FC events include the presence of Michel electrons, discussed in more detail in Subsection 3.5.1, as well as studies of the probability that π^0 events are misidentified as e-like, which are described later in this chapter.

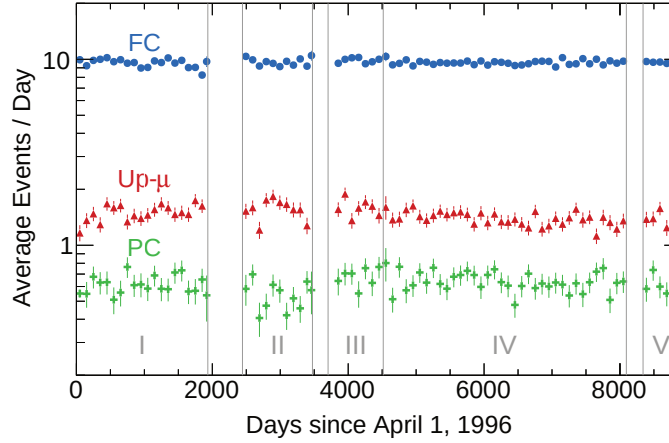


Figure 3.10. Final event rates as a function of time since the start of SK operations. The error bars represent statistical uncertainties. Blue circles indicate fully contained (FC) event rates, red upward-pointing triangles represent upward-going muon ($\text{Up-}\mu$) event rates, and green squares denote partially contained (PC) event rates. Figure taken from [93].

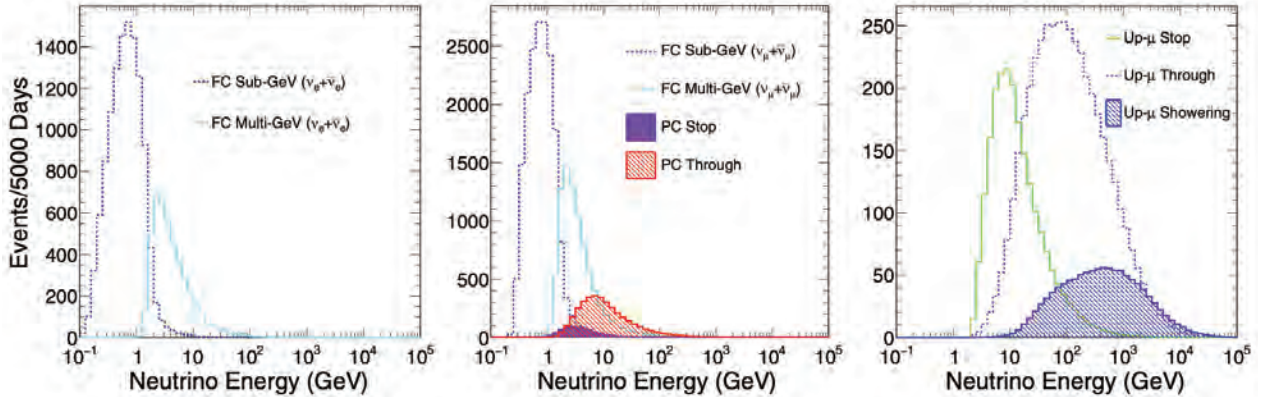


Figure 3.11. True atmospheric neutrino energy spectra from simulation in the SK detector for the three event categories: fully contained (FC), partially contained (PC), and upward-going muons ($\text{Up-}\mu$), as described in the text. Note that neutrino oscillations are not included in these simulated spectra. Figure taken from [94].

The FC Sub-GeV single-ring e-like sample contains background events primarily originating from neutral-current (NC) interactions where π^0 was produced, and in which one of the two γ rays from the π^0 decay is not reconstructed. The electromagnetic shower produced by the remaining γ ray generates a light pattern similar to that of an electron, resulting in an e-like classification. To reduce this background, a dedicated π^0 fitter implemented in APFIT is used [135]. This fitter forces a second ring in the event reconstruction and predicts the corresponding light pattern expected from two γ rays propagating through the detector, given the fitted vertex and ring directions. The intensities and directions of both fitted rings are varied to achieve the best agreement between the predicted and observed light distributions. Since the signal process of interest—charged-current quasi-elastic (CCQE) interactions—produces only one visible particle (the proton being mostly below Cherenkov threshold for FC Sub-GeV single-ring events), the invariant mass constructed from the two fitted rings serves as an effective variable to distinguish CCQE signal events from NC π^0 background.

For the categorization of Multi-GeV multi-ring events, a boosted decision tree (BDT) classifier is employed. This method enables the separation of events into four categories: ν_e -like, $\bar{\nu}_e$ -like, μ -like, and “other.” The “other” category primarily contains neutral-current (NC) interactions. Further details of the BDT implementation and its performance are provided in Subsection 3.5.2 on page 61.

Partially contained (PC) events typically have neutrino energies between 1 GeV and 1 TeV (see Fig. 3.11), and consist almost entirely of ν_μ charged-current (CC) interactions, as the produced muon frequently exits the inner detector (ID). PC events are further classified as OD “stopping” or OD “through-going” based on the amount of energy deposited in the outer detector (OD). Currently the separation between these two categories is achieved by comparing the amount of light recorded in the OD with expectations from simulated PC events. However, additional studies aiming to improve the classification of the PC sample are ongoing and they include adding information on the decay electrons available in the OD region only for events in which the muon decayed inside OD.

The Up- μ sample consists of upward-going muon events produced by neutrino interactions in the rock surrounding SK or in the water of the outer detector (OD). These represent the highest-energy events observed at SK, with energies reaching up to ~ 10 TeV (see Fig. 3.11). The Up- μ events are categorized into three classes: “stopping,” if the muon stops within the inner detector (ID); or “showering” and “non-showering,” depending on whether the exiting muon’s energy loss pattern is consistent with significant radiative processes.

3.5.1. Separation of the neutrino and antineutrino interactions at the SK

In the SK detector, ν_e and $\bar{\nu}_e$ interactions—crucial for neutrino mass ordering studies—cannot be distinguished on an event-by-event basis because the detector has no magnetic field. The absence of a magnetic field is necessary to avoid interference with the performance of the photomultiplier tubes (PMTs) (see Section 2.1.2 on page 29). Consequently, for example, in ν_e and $\bar{\nu}_e$ interactions, it is not possible to differentiate between the Cherenkov rings produced by e^- and e^+ . However, a statistical separation between ν_e and $\bar{\nu}_e$ interactions can be achieved using additional event information described below. Since ν_e interactions have both larger cross sections and higher fluxes than $\bar{\nu}_e$ interactions, more than twice as many ν_e events are expected in the data.

For example, in the processes:

$$\bar{\nu}_e + p \rightarrow e^+ + p + \pi^- \quad (3.22)$$

$$\bar{\nu}_e + n \rightarrow e^+ + n + \pi^- \quad (3.23)$$

$$\nu_e + p \rightarrow e^- + p + \pi^+ \quad (3.24)$$

$$\nu_e + n \rightarrow e^- + n + \pi^+ \quad (3.25)$$

with $\bar{\nu}_e$ interactions with protons or neutrons, the resulting negatively charged pion is likely (approximately 80%) to be captured by a ^{16}O nucleus before it decays. This capture process does not occur for the positively charged pion produced in the corresponding ν_e interactions. Captured pions do not yield Michel electrons from the decay chain of an unseen charged pion (referred to as decay electrons, or *d.e.*). Consequently, requiring one or more decay electrons preferentially selects neutrino interactions over antineutrino interactions for this process.

After applying the π^0 -like event rejection, the remaining e-like sample is divided into two categories based on the number of decay electrons: a “0-decay” sample, which contains no decay electrons originating from μ decay, and a “1-decay” sample, which includes events with one or more decay electrons. In ν_e CCQE interactions, no decay electrons are expected, and therefore a large fraction of ν_e CCQE interactions populate the e-like FC Sub-GeV single-ring 0-decay sample.

For the FC Sub-GeV single-ring μ -like events—crucial for the $\nu_\mu \leftrightarrow \nu_\tau$ (*disappearance*) oscillation channel—three categories are defined based on the number of decay electrons: 0-decay, 1-decay, and 2-decay. Since CCQE interactions in this case produce a μ^- , at least one decay electron is typically expected. The statistical separation can be further improved by also considering the number of neutrons, which will be described further.

SK-IV and SK-V neutrons captured on hydrogen

Starting from SK-IV, information from neutrons captured on hydrogen has been recorded and used for the first time in this version of the atmospheric neutrino analysis. Detecting neutrons in SK is experimentally challenging, but it has the potential to improve the sensitivity of several analyses [136].

For example, neutron detection can enhance the statistical separation between neutrino and antineutrino interactions, since neutrino events are expected to produce fewer neutrons than antineutrino events (this is generally valid in the $\sim$ GeV energy region, while at higher energies additional processes become significant). The clearest case is that of charged-current quasi-elastic (CCQE) interactions, where $\bar{\nu}_e$ interactions produce a neutron in the final state, whereas ν_e interactions produce a proton:

$$\nu_e + n \rightarrow e^- + p \quad (3.26)$$

$$\bar{\nu}_e + p \rightarrow e^+ + n \quad (3.27)$$

The neutrons produced in Eq. (3.27) are quickly thermalized. On average, it takes about twenty collisions with free protons in water over a period of approximately $10 \mu\text{s}$ to reduce the neutron emitted with a few MeV of kinetic energy down to thermal equilibrium ($\sim 0.025 \text{ eV}$). Once thermalized, the neutron continues to scatter for another $\sim 100 \mu\text{s}$ before being captured by either a hydrogen or oxygen nucleus in the water. The capture cross sections for these reactions are 0.33 barns and 0.19 mbarns, respectively. Thus, to first approximation, nearly all thermal neutrons are captured on hydrogen via the reaction:

$$n + p \rightarrow d + \gamma \quad (3.28)$$

The de-excitation of the captured deuteron produces a 2.2 MeV γ ray, which generates a small cluster of time-coincident PMT hits localized in space. This γ ray can scatter electrons in the water via Compton scattering, accelerating some of them above the Cherenkov threshold. Detecting the Cherenkov light from these electrons allows inference of the γ ray's presence and, consequently, the capture of its parent neutron.

However, identifying 2.2 MeV γ rays from neutron captures on hydrogen is challenging. The conventional event reconstruction threshold at SK is approximately 4 MeV, corresponding to about 34 PMT hits, whereas a 2.2 MeV γ typically produces only ~ 7 hits. Because of this limitation, full reconstruction of neutron captures on hydrogen is not feasible. Instead, a dedicated two-step neutron tagging algorithm has been developed.

In the first step, a sliding 10 ns hit-time window is used to search for candidate neutron captures by identifying clusters containing 7–50 hits, provided that fewer than 200 hits occur within a surrounding 200 ns window. The lower hit threshold suppresses random noise coincidences, while the upper threshold reduces contamination from decay electrons. In the second step, several variables are calculated for each candidate cluster, including measures of hit isotropy, the likelihood of a single-vertex origin, and the temporal spread of the hits. These variables are then used as inputs to a neural network that classifies each candidate as either signal or background. When applied to SK-IV,V atmospheric neutrino MC events, the neural network achieves an average neutron-tagging efficiency for neutron captures on hydrogen of 26 %, with a background rate of 0.016 false neutrons per event.

The uncertainty in the neutron-tagging efficiency is evaluated using an americium–beryllium (AmBe) calibration source embedded in a scintillating box and deployed at various positions throughout the detector. The systematic uncertainty on the efficiency is estimated to be 9 %. A detailed description of the neutron-tagging algorithm and its development can be found in [137].

The number of tagged neutrons, together with the information on decay electrons, is used as an additional input in the classification of fully contained (FC) single-ring events from the SK-IV-V

phases. The modifications to the event selections are identical for both Sub-GeV and Multi-GeV samples and account for the fact that neutrino charged-current (CC) interactions typically produce fewer neutrons than antineutrino CC interactions. Consequently, dedicated selection criteria have been developed separately for the e-like and μ -like samples.

For e-like events, there are three sub samples:

- If the event has at least one decay electron, it is classified as ν_e -like.
- If the event has no decay electrons and no tagged neutrons, it is classified as a ν_e - $\bar{\nu}_e$ -like mixture.
- If the event has no decay electrons at least one tagged neutron, it is classified as $\bar{\nu}_e$ -like.

For μ -like events, there are two sub samples:

- If the event has exactly one decay electron, and at least one tagged neutron, it is classified as $\bar{\nu}_\mu$ -like.
- Otherwise, it is classified as a ν_μ -like.

Future extensions: neutrons captured on gadolinium

Detecting neutron captures on hydrogen in SK is challenging. Since 2020, the ultra-pure water in the detector has been enriched with gadolinium sulfate to enhance neutron detection efficiency. Gadolinium has an exceptionally large thermal neutron absorption cross section of 49,700 barns and has long been used in scintillator-based neutrino experiments near nuclear reactors. Metallic gadolinium, however, is essentially insoluble in water and therefore unsuitable for use in a large water Cherenkov detector. In contrast, gadolinium sulfate, $\text{Gd}_2(\text{SO}_4)_3$, is highly soluble. Neutron capture on gadolinium results in a cascade of γ rays, producing a 7.9 MeV cascade 80.5% of the time and an 8.5 MeV cascade 19.3% of the time. This energy release is easily detectable in SK. The addition of gadolinium is expected to improve significantly the statistical separation of $\bar{\nu}_e$ and ν_e events in future analyses of atmospheric (anti)neutrino oscillations at SK.

3.5.2. Multi-GeV neutrino events

Multi-GeV events are classified as single-ring or multi-ring samples, depending on the number of reconstructed Cherenkov rings. For single-ring events, the particle identification (PID) of the ring is used to further divide the sample into e-like and μ -like categories.

In SK-I-III, Multi-GeV single-ring e-like events are separated into two sub samples based on the number of decay electrons: events with zero decay electrons are categorized as $\bar{\nu}_e$ -like, while those with one or more decay electrons are categorized as ν_e -like. For SK-IV-V, the $\bar{\nu}_e$ -like sample is further refined using neutron-tagging information. Events with no tagged neutrons are classified as e-like (mixed), whereas events with one or more tagged neutrons are identified as $\bar{\nu}_e$ -like (see Fig. 3.14 and Fig. 3.15).

In the case of SK-I-III, no further subdivision is applied to the Multi-GeV single-ring μ -like events based on the number of decay electrons. However, for SK-IV-V, the μ -like sample is further divided into $\bar{\nu}_\mu$ -like and ν_μ -like categories. Events with one decay electron and one or more tagged neutrons are classified as $\bar{\nu}_\mu$ -like, whereas events with a decay-electron count different from one, or with zero tagged neutrons, are categorized as ν_μ -like. At neutrino energies of a multi-GeV and above, events are predominantly produced through deep inelastic scattering (DIS) interactions. These processes often generate multiple pions along with nuclear fragments in the final state. To purify and classify the resulting multi-ring samples, seven reconstructed observables—illustrated in Fig. 3.12 and described in [138]—are used.

As in the single-ring classification, the primary variable for distinguishing μ -like from e-like events is the particle identification (PID) information of the most energetic ring in each event. To further enhance

e - μ separation, we additionally consider the maximum distance between any decay electron and the reconstructed event vertex, denoted as $L_{\text{Decay}e}$. This variable shows characteristic differences between μ -like and e -like events. Specifically, in muon-neutrino CC interactions, decay electrons originating from muon decays tend to occur at larger distances due to the higher momentum of the parent muon, whereas in electron-neutrino CC interactions, muons from charged-pion decays typically have lower momenta and therefore shorter decay-electron distances. To distinguish ν_e -like from $\bar{\nu}_e$ -like events, we again utilize the number of decay electrons, complemented by additional kinematic information available in multi-ring events. In particular, we consider the number of reconstructed rings and two variables that describe the event's momentum distribution: the fraction of total momentum carried by the most energetic ring, $p_{\text{Max.}}/p_{\text{tot.}}$, and the fraction of momentum transverse to the direction of the most energetic ring, $p_T/p_{\text{tot.}}$. These quantities differ systematically between ν_e and $\bar{\nu}_e$ events due to the distinct angular and momentum dependencies of their respective cross sections. Furthermore, the visible energy is used to help identify ν_τ events, which only appear at higher energies due to the τ production threshold ($E_\tau \geq 3.5$ GeV). This analysis replaces the likelihood-based selection employed in previous studies [94] with a boosted decision tree (BDT) classifier that uses the same set of variables (see Fig. 3.12) as inputs. The most significant differences between electron- and muon-flavor neutrino interactions arise from the PID of the most energetic ring and the maximum distance traveled by decay electrons, $L_{\text{Decay}e}$. Electron-neutrino and electron-antineutrino multi-ring events, on the other hand, differ primarily in the expected number of decay electrons, the number of reconstructed rings, and in the fractional momentum variables $p_{\text{Max.}}/p_{\text{tot.}}$ and $p_T/p_{\text{tot.}}$. Fig. 3.13 illustrates the ν_e - $\bar{\nu}_e$ separation performance of the BDT for the SK-IV,V phases. Neutron information from the SK-IV,V phases was also considered as an input variable to the BDT, but was found to only result in a marginal improvement to ν_e - $\bar{\nu}_e$ separation while introducing a large amount of systematic uncertainty.

Based on the discussed variables and the BDT output, four analysis samples are defined for multi-ring events: CC ν_e -like, CC $\bar{\nu}_e$ -like, CC μ -like (including both ν_μ and $\bar{\nu}_\mu$), and “other.” The

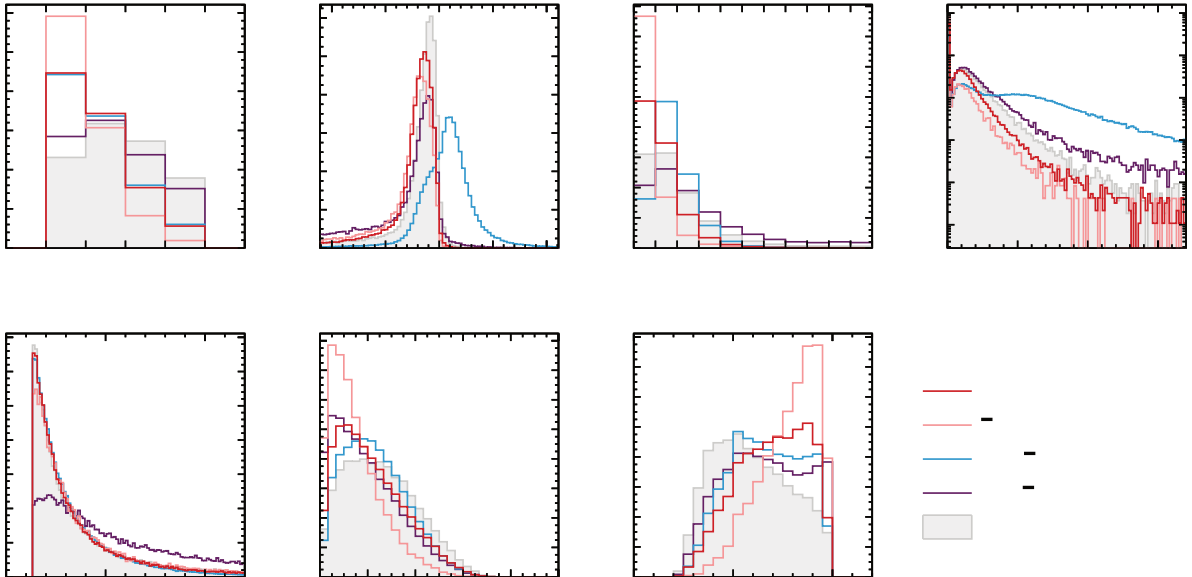


Figure 3.12. Area-normalized SK-IV-V MC distributions of the different input variables used for the FC Multi-Ring BDT selection. True CC neutrino interactions are shown as solid lines while the NC contribution is shown as filled histograms. Figure taken from [93].

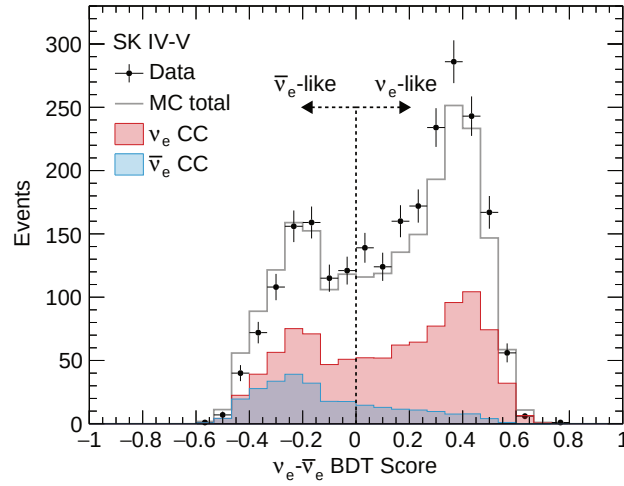


Figure 3.13. ν_e and $\bar{\nu}_e$ separation using the Multi-Ring boosted decision tree (BDT) classification for SK-IV-V data and MC events. The BDT assigns each event a final classification according to the class with the highest output score. Negative BDT values correspond to $\bar{\nu}_e$ -like events, while positive values correspond to ν_e -like events. The shown distributions correspond to the SK-IV,V data-taking periods and include events reconstructed within the expanded fiducial volume region. The “MC total” spectrum represents the sum of all simulated contributions, including true ν_μ , ν_τ , and neutral-current (NC) interactions. Figure taken from [93].

“other” category is designed to isolate neutral-current (NC) and ν_τ interactions from the CC-enhanced samples.

3.5.3. Summary of the atmospheric neutrino selections

Overall, all FC events are divided into 14 sub samples for SK-I-III referred to as the *standard classification*, (note that the Multi-Ring μ -like sample presented in Fig. 3.14 with Sub-GeV and Multi-GeV range is combined into one in the analysis). 16 sub samples are used for SK-IV-V, where neutron-tagging information is additionally incorporated. The sub samples are defined according to the visible energy in the detector (E_{vis}), the number of reconstructed Cherenkov rings, the particle identification

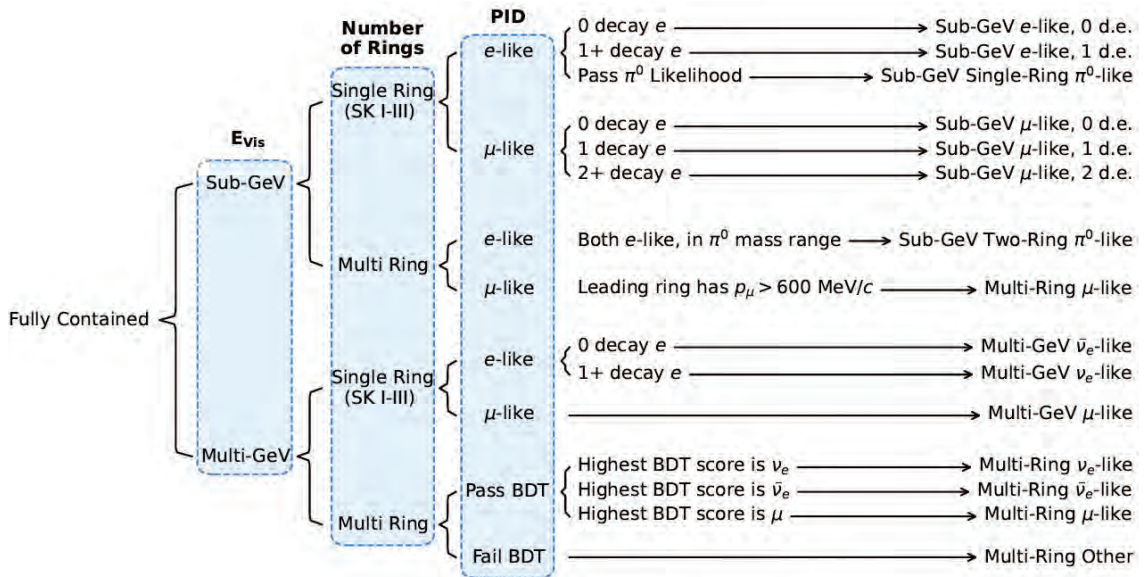


Figure 3.14. Overview of the standard FC events selections without using information on neutron tagging. Figure taken from [139].

(PID) information, and the numbers of decay electrons and tagged neutrons. Fig. 3.14 provides an overview of the standard FC sub-sample selection, excluding the neutron-tagging information.

Fig. 3.15 illustrates the modified event selection scheme used in SK-IV-V, which enhances the statistical separation between ν_e and $\bar{\nu}_e$ events by incorporating neutron-tagging information, in contrast to the standard selection applied in SK-I-III.

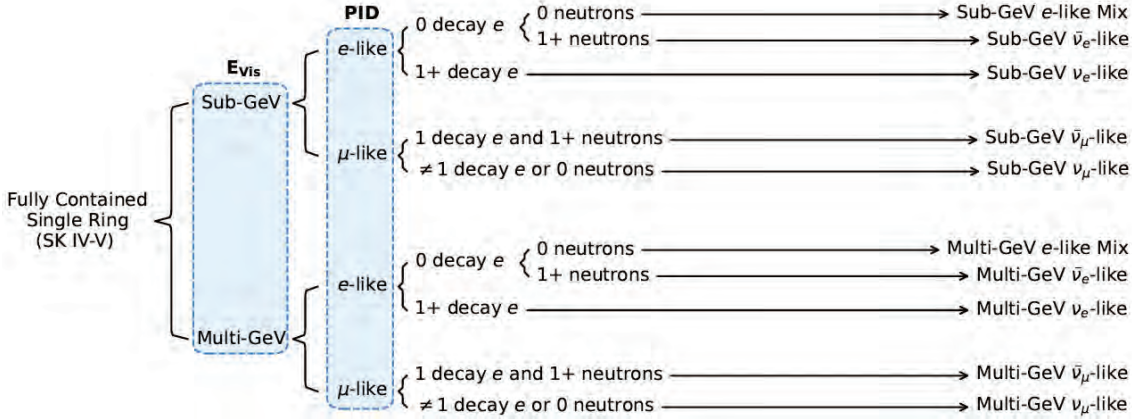


Figure 3.15. Overview of the modified FC event selections for SK-IV-V using neutron tagging information. Figure taken from [139].

The event classification discussed here corresponds to 19 analysis samples for SK-I-III and 21 samples for SK-IV-V. The data and MC events for the 19 standard analysis samples are shown in Fig. 3.16, while the modified selections using neutron capture information are presented in Fig. 3.17, both as functions of reconstructed lepton momentum or zenith angle. Table 3.2 summarizes the Monte Carlo CC and NC purities, as well as the corresponding data event counts, for each sample used in the analysis.

Notably, the SK-IV-V Multi-GeV $\bar{\nu}_e$ sample, which employs neutron tagging, exhibits a higher purity of true $\bar{\nu}_e$ CC events compared to the corresponding SK-I-III sample. Improvements in selection purity are also evident in the multi-ring classification obtained using the BDT method.

Compared to the likelihood-based classification employed in the previous analysis [94], the optimized BDT significantly enhances the efficiency of correctly identifying true CC events. The efficiency increases from 35 % to 55 % for ν_e , from 58 % to 65 % for $\bar{\nu}_e$, and from 74 % to 81 % for ν_μ , while the efficiency for correctly classifying NC events decreases from 57 % to 31 %.

The purity of true ν_e CC events in the ν_e -like sample decreases slightly from 56 % to 50 %, whereas the purities of the $\bar{\nu}_e$ -like and ν_μ -like samples remain unchanged. Moreover, the purity of correctly classified NC events in the “other” sample increases from 30 % to 43 %. After accounting for exposure differences, the improved CC classification efficiencies lead to proportionally larger event statistics in the ν_e -like, $\bar{\nu}_e$ -like, and ν_μ -like multi-ring samples, accompanied by a corresponding reduction in the “other” sample compared to the previous analysis.

3.6. Atmospheric neutrino oscillation analysis method

The neutrino oscillation parameters are derived from the two independent fitting approaches. In the first and least-constrained fit, the SK atmospheric neutrino data are fit allowing θ_{13} to vary as a free parameter. The second fit similarly uses only atmospheric neutrino data, but assumes θ_{13} to be the average of several reactor neutrino disappearance experiment measurements, $\sin^2 \theta_{13} = 0.0220 \pm 0.0007$ [26].

Table 3.2. This analysis summarizes the Monte Carlo (MC) charged-current (CC) and neutral-current (NC) purities for each event sample, together with the corresponding data events. The reported purities and MC expectations are evaluated with neutrino oscillation probabilities applied, but without the inclusion of systematic uncertainty pulls. The “ ν_τ CC” column denotes the combined purity of ν_τ and $\bar{\nu}_\tau$ charged-current interactions. The Up- μ data are presented after background subtraction. Taken from [93].

Sample	Energy bins	$\cos\theta_z$ bins	MC purity						Events	
			ν_e CC	$\bar{\nu}_e$ CC	ν_μ CC	$\bar{\nu}_\mu$ CC	ν_τ CC	NC	MC	Data
<i>Fully contained (FC), single ring, Sub-GeV</i>										
SK I-III										
e-like										
0 decay-e	5 $e^\pm$ momentum	10 in $[-1, 1]$	0.733	0.226	0.003	0.001	0.000	0.036	6409.1	6647
1 decay-e	5 $e^\pm$ momentum	single bin	0.796	0.016	0.086	0.020	0.001	0.081	612.0	682
μ -like										
0 decay-e	5 $\mu^\pm$ momentum	10 in $[-1, 1]$	0.027	0.008	0.704	0.149	0.001	0.112	2153.9	2419
1 decay-e	5 $\mu^\pm$ momentum	10 in $[-1, 1]$	0.001	0.000	0.677	0.291	0.000	0.030	4241.4	4476
2 decay-e	5 $\mu^\pm$ momentum	single bin	0.001	0.000	0.948	0.029	0.001	0.022	330.7	336
SK IV-V										
ν_e -like	5 $e^\pm$ momentum	10 in $[-1, 1]$	0.794	0.016	0.090	0.024	0.001	0.074	943.7	1093
$\bar{\nu}_e$ -like 0 n	5 $e^\pm$ momentum	10 in $[-1, 1]$	0.789	0.175	0.003	0.001	0.000	0.031	5961.8	6669
$\bar{\nu}_e$ -like 1 n	5 $e^\pm$ momentum	10 in $[-1, 1]$	0.582	0.367	0.003	0.002	0.001	0.044	2266.6	1668
ν_μ -like	5 $\mu^\pm$ momentum	10 in $[-1, 1]$	0.011	0.003	0.755	0.173	0.000	0.057	6596.0	7879
$\bar{\nu}_\mu$ -like	5 $\mu^\pm$ momentum	10 in $[-1, 1]$	0.001	0.000	0.533	0.417	0.001	0.049	2150.4	1793
<i>Fully contained (FC), single ring, Multi-GeV</i>										
SK I-III										
ν_e -like	4 $e^\pm$ momentum	10 in $[-1, 1]$	0.568	0.086	0.102	0.014	0.039	0.190	359.6	383
$\bar{\nu}_e$ -like	4 $e^\pm$ momentum	10 in $[-1, 1]$	0.556	0.341	0.013	0.002	0.012	0.075	1359.8	1339
ν_μ -like	2 $\mu^\pm$ momentum	10 in $[-1, 1]$	0.002	0.001	0.621	0.371	0.003	0.002	1588.5	1564
SK IV-V										
ν_e -like	4 $e^\pm$ momentum	10 in $[-1, 1]$	0.607	0.087	0.098	0.015	0.033	0.159	584.1	643
$\bar{\nu}_e$ -like 0 n	4 $e^\pm$ momentum	10 in $[-1, 1]$	0.637	0.287	0.008	0.002	0.007	0.058	866.4	986
$\bar{\nu}_e$ -like 1 n	4 $e^\pm$ momentum	10 in $[-1, 1]$	0.435	0.460	0.009	0.002	0.015	0.079	736.1	616
ν_μ -like	2 $\mu^\pm$ momentum	10 in $[-1, 1]$	0.002	0.001	0.695	0.297	0.003	0.001	1464.0	1619
$\bar{\nu}_\mu$ -like	2 $\mu^\pm$ momentum	10 in $[-1, 1]$	0.001	0.000	0.446	0.549	0.004	0.002	593.1	503
SK I-V common samples										
<i>Fully contained (FC) Sub-GeV NC π^0-like</i>										
Single-ring	5 $e^\pm$ momentum	single bin	0.219	0.064	0.018	0.002	0.001	0.696	748.0	868
Two-ring	5 π^0 momentum	single bin	0.096	0.028	0.015	0.001	0.000	0.860	2095.3	2494
<i>Fully contained (FC) Multi-GeV, multi-ring</i>										
ν_e -like	3 visible energy	10 in $[-1, 1]$	0.495	0.066	0.175	0.014	0.035	0.215	2149.7	2411
$\bar{\nu}_e$ -like	3 visible energy	10 in $[-1, 1]$	0.519	0.260	0.052	0.007	0.025	0.138	1210.0	1131
μ -like	4 visible energy	10 in $[-1, 1]$	0.028	0.003	0.713	0.200	0.006	0.050	3257.7	3427
Other	4 visible energy	10 in $[-1, 1]$	0.203	0.023	0.257	0.014	0.074	0.429	837.9	982
<i>Partially-contained (PC)</i>										
Stopping	2 visible energy	10 in $[-1, 1]$	0.089	0.034	0.559	0.262	0.011	0.045	641.6	689
Through-going	4 visible energy	10 in $[-1, 1]$	0.006	0.002	0.638	0.341	0.007	0.006	3310.2	3397
<i>Upward-going muons (Up-μ)</i>										
Stopping	3 visible energy	10 in $[-1, 0]$	0.008	0.003	0.646	0.340	0.000	0.003	1574.3	1753.8
Non-showering	single bin	10 in $[-1, 0]$	0.002	0.001	0.662	0.334	0.000	0.001	5315.8	6423.9
Showering	single bin	10 in $[-1, 0]$	0.001	0.000	0.671	0.327	0.000	0.001	1051.4	1110.6

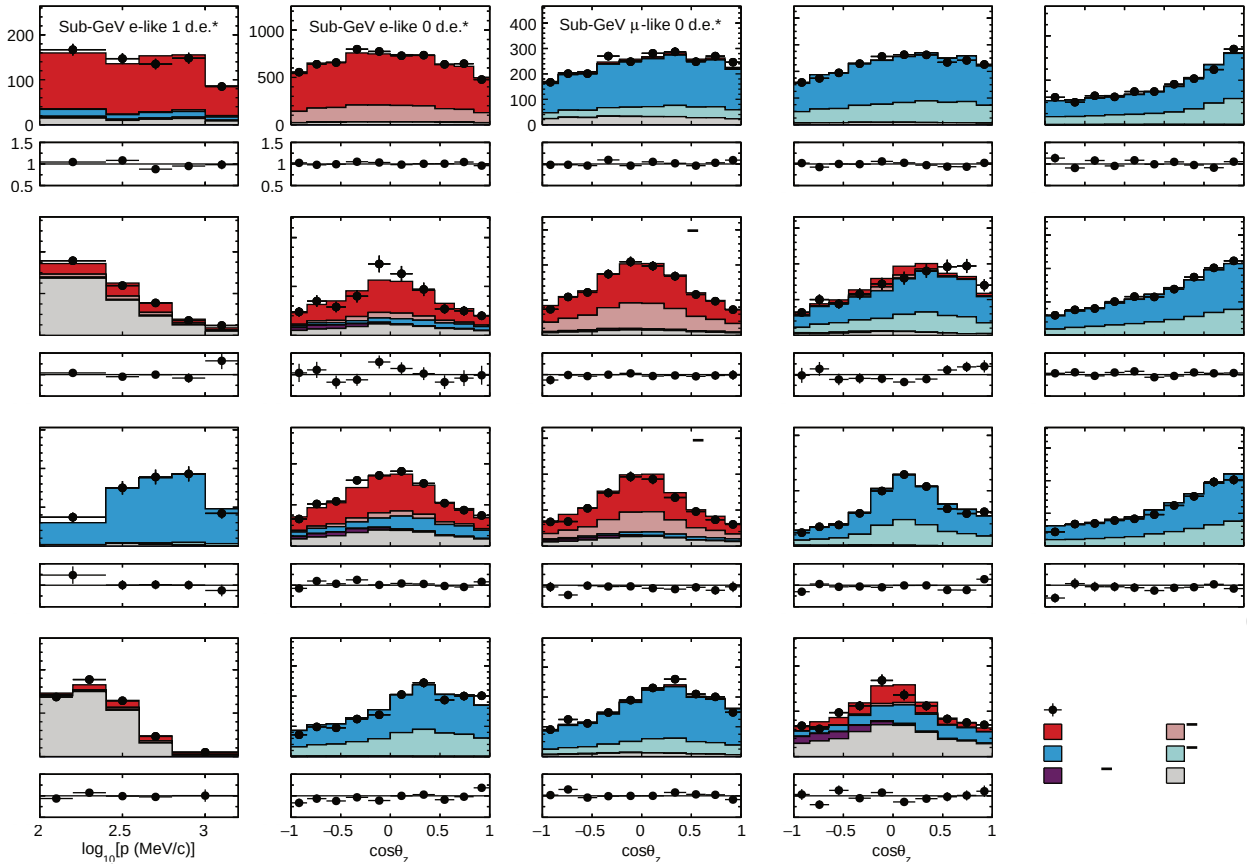


Figure 3.16. Zenith-angle or momentum distributions for the 19 standard analysis samples without neutron tagging are shown. Black data points represent the observed number of events in each bin, while the filled histograms indicate the expected event contributions from the MC prediction. Different colors in the MC histograms correspond to the true neutrino flavors contributing to each sample. The lower panels in each distribution display the data-to-MC ratio, with all error bars representing statistical uncertainties. Fully contained (FC) Sub-GeV and Multi-GeV single-ring samples, marked with an asterisk (*), include events only from SK-I-III, while the corresponding events from SK-IV-V are separated into sub samples using neutron-tagging information, as shown in Fig. 3.17. Figure taken from [93].

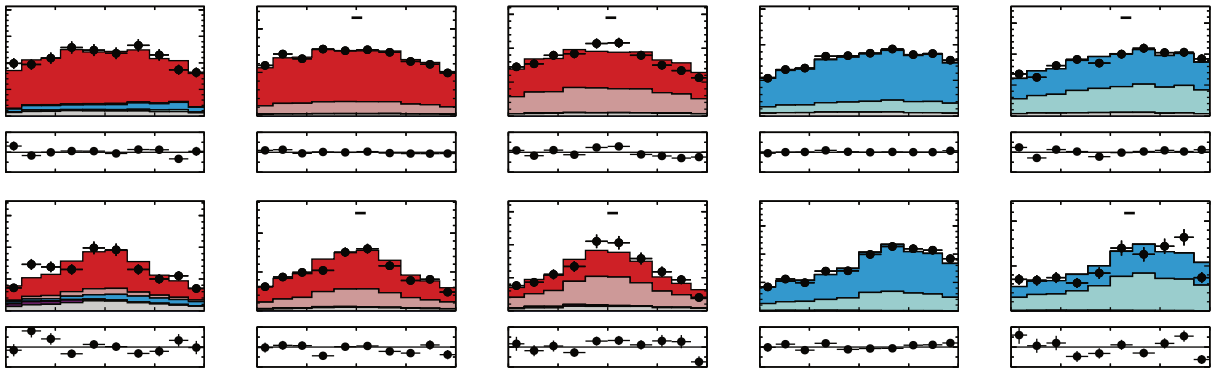


Figure 3.17. Zenith angle distributions for the SK-IV & SK-V FC Sub-GeV and Multi-GeV single-ring samples using the number of tagged neutrons. The MC configuration and the meaning of the colors are the same as in Fig. 3.16. Figure taken from [93].

In both analyses, the agreement between data and Monte Carlo (MC) predictions is quantified by evaluating a χ^2 statistic at each point on a fixed grid of neutrino oscillation parameters, as defined

Table 3.3. The oscillation parameter grid definitions used in the analyses presented in this work are summarized here. Each grid is uniformly spaced and includes both the minimum and maximum parameter values. The same grid configurations are applied for both the normal and inverted mass orderings across all analyses. In both analyses, the parameters Δm_{21}^2 and $\sin^2 \theta_{12}$ are treated as constrained quantities. Taken from [93].

Parameter	Min.	Max.	Points
SK, Atmospheric Only			
$\sin^2 \theta_{23}$	0.3	0.775	20
Δm_{32}^2 (10^{-3} eV^2)	1.2	3.6	25
δ_{CP} (rad)	$-\pi$	π	21
$\sin^2 \theta_{13}$	0	0.075	16
SK, $\sin^2 \theta_{13}$ Constrained			
$\sin^2 \theta_{23}$	0.3	0.7	35
Δm_{32}^2 (10^{-3} eV^2)	1.0	4.9	40
δ_{CP} (rad)	$-\pi$	π	37
$\sin^2 \theta_{13}$	0.0220 ± 0.0007		1

in Table 3.3. For every grid point, oscillation probabilities are applied to the MC samples using the corresponding parameter values, after which the systematic uncertainty nuisance parameters are fitted to obtain the minimum χ^2 value.

The χ^2 statistic is computed as a sum over n bins, assuming Poisson statistics and incorporating systematic uncertainty pull terms, ϵ_i , each expressed in units of σ . Each ϵ_i represents the fitted scaling factor for the i^{th} systematic uncertainty and contributes to the χ^2 calculation as an additional penalty term [93]:

$$\chi^2 = 2 \sum_n \left(E_n - O_n + O_n \ln \frac{O_n}{E_n} \right) + \sum_i \epsilon_i^2 \quad (3.29)$$

In Eq. (3.29), n indexes each bin and O_n are the observed counts in the n^{th} bin. The expected counts E_n are calculated from a nominal MC prediction E_n^0 scaled by the effect of systematic uncertainties. The effect of the i^{th} systematic uncertainty is calculated as ϵ_i times the fractional change induced in the n^{th} bin by a 1σ change, $f_{i,n}$:

$$E_n = E_n^0 \left(1 + \sum_i f_{i,n} \epsilon_i \right) \quad (3.30)$$

The best agreement between data and MC is obtained by solving the system of equations defined by the condition $\partial \chi^2 / \partial \epsilon_i = 0$ for each systematic uncertainty parameter. In other words, enforcing $\partial \chi^2 / \partial \epsilon_i = 0$ yields the set of ϵ_i values that minimize χ^2 for a given MC prediction. This minimization over the systematic uncertainty parameters is performed at each point on the oscillation parameter grid (see Table 3.3), and the parameter set that produces the lowest χ^2 value is taken as the best fit.

3.6.1. Systematic uncertainties

This section summarizes the systematic uncertainties considered in this analysis and the corresponding fractional change in ϵ_i when each is varied by $\pm 1\sigma$. In total, the analysis includes 193 independent sources of systematic uncertainty. These can be grouped into several categories: uncertainties related

to the atmospheric neutrino flux and interaction cross sections (48 sources common to all SK phases), and reconstruction efficiency uncertainties (29 sources per phase, estimated independently for each of the five SK data-taking periods).

The largest contributions, and those with the strongest impact on the fitted oscillation parameters, arise from the flux and cross-section modeling. However, detector performance effects can also introduce non-negligible biases in certain oscillation parameters, as discussed below. In the following, we highlight the most significant systematic sources included in this analysis.

Atmospheric neutrino flux uncertainties

Several uncertainties in the atmospheric neutrino flux are estimated by comparing the flux predictions of the Honda model [111] with those from two alternative models, Bartol [102] and FLUKA [101], as discussed in Section 3.2.1 (page 48). The atmospheric neutrino flux uncertainties account for variations in the overall flux normalization, flavor ratios, K/π production ratio, neutrino path length, up/down asymmetry, and the effects of solar activity. The following paragraphs describe several of these sources in more detail, beginning with the first category.

Absolute normalization. The overall flux normalization uncertainty is estimated from the combined uncertainties in the hadron production models, air density, and hadron interaction models used as inputs to the atmospheric neutrino flux calculation. The magnitude of this uncertainty is illustrated in Fig. 3.18, and it is applied by varying the event weights independently for neutrino energies below and above 1 GeV.

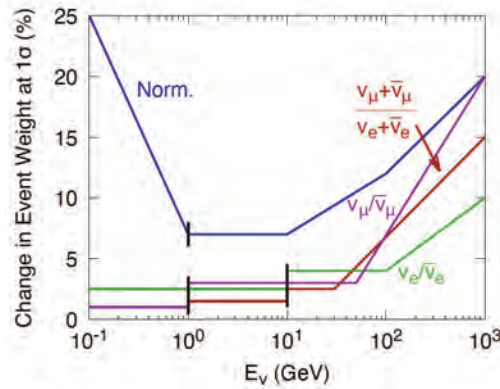


Figure 3.18. Fractional change in the event weight for a 1σ change in the absolute normalization uncertainty and the flavor ratio uncertainties of the flux. Figure taken from [139].

Flavor ratios. Three sets of uncertainties are considered for the overall muon-to-electron flavor ratio, $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$, as well as for the individual neutrino–antineutrino ratios, $\nu_\mu/\bar{\nu}_\mu$ and $\nu_e/\bar{\nu}_e$. These flavor-ratio uncertainties are evaluated by comparing the flux predictions from three different models: the Honda model [99], the FLUKA model [101], and the Bartol model [102]. The magnitudes of these uncertainties are shown in Fig. 3.18. Because the dominant sources of uncertainty vary across energy ranges, each uncertainty is divided into three independent components corresponding to $E_\nu < 1$ GeV, $1 < E_\nu < 10$ GeV, and $E_\nu > 10$ GeV.

K/π ratio. Atmospheric neutrinos are primarily produced in pion decays; however, above 10 GeV, the contribution from kaon decays becomes non-negligible. The uncertainty associated with the $K^\pm$ production ratio is evaluated by varying this ratio in the Honda flux calculation. The resulting fractional

change in event weights is approximately 3% in the highest energy bins of the FC and PC samples, and about 10% in the Up- μ samples.

Neutrino path length. For neutrinos originating from above the horizon, the neutrino path length is determined by the production height, which depends on the atmospheric density profile. The effect of a 10% variation in the density profile on the oscillation probability is included as a source of systematic uncertainty.

Up/down ratio. The geomagnetic field introduces an up/down asymmetry in the low-energy atmospheric neutrino flux. The uncertainty in the magnitude of this asymmetry is evaluated by comparing predictions from the three flux models. The resulting effect on the event weights is approximately 3% in the most affected energy bins.

Horizontal/vertical ratio. The uncertainty in the horizontal-to-vertical flux ratio arises primarily from the three-dimensional atmospheric neutrino flux calculation. This uncertainty is evaluated by comparing predictions from the three flux models, and its effect on the event weights is approximately 2.5%.

Relative normalization. While the absolute normalization uncertainty accounts for the overall flux predicted by the Honda model, differences in the energy dependence of the flux among the three models introduce an additional uncertainty. This uncertainty is intended to cover the discrepancies observed in the high-energy region. A 5% normalization uncertainty is assigned to the FC Multi-GeV, PC, and Up- μ stopping samples. The effects on the PC and Up- μ samples are treated as correlated, while those on the FC samples are considered independent of PC+Up- μ .

Solar activity. As described in Section 3.2.1 on page 48, the predicted flux is weighted to account for variations in solar activity during each phase. The uncertainty in these weights is evaluated by shifting the time period used to determine the weights from the neutron data by ± 1 year. This uncertainty is assessed separately for each phase, and the corresponding error parameters are allowed to vary independently during the fit. The resulting effect on the event weights is largest at low energies, reaching approximately 3% in the most affected bins.

Neutrino interaction uncertainties

This analysis implements 26 uncertainties for various aspects of the cross section models used in NEUT (see Section 3.2 on page 48).

Quasi-elastic uncertainties. Quasi-elastic (QE) uncertainties modify parameters within both the 1p1h and 2p2h cross-section models. These uncertainties primarily affect CC 1p1h, or CCQE, interactions. For these events, the NEUT default model—Local Fermi Gas (LFG) with Random Phase Approximation (RPA) corrections and $M_A^{QE} = 1.05 \text{ GeV}/c^2$ —is compared to an alternative Relativistic Fermi Gas (RFG) model without RPA corrections and with $M_A^{QE} = 1.21 \text{ GeV}/c^2$. Throughout this section, these are referred to as the “nominal LFG model” and the “RFG model,” respectively.

Quasi-elastic axial mass. The uncertainty on M_A^{QE} , introduced in Eq. (3.5), affects the axial form factor, thereby modifying both the total cross section and the q^2 dependence of the CCQE interaction model. This uncertainty is evaluated by reweighting CCQE events according to the double-differential cross-section ratio between a 1σ variation of M_A^{QE} and its nominal value. The 1σ variation corresponds to $M_A^{QE} = 1.21 \text{ GeV}/c^2$. The resulting fractional difference between the two models is taken as the

systematic uncertainty, with its effect on the event weights reaching up to 10% in the most affected kinematic regions.

CCQE shape. This uncertainty quantifies the relative difference in the neutrino energy dependence between the RFG and LFG models. The 1σ variation is defined as the fractional difference in the normalized total cross sections, as a function of E_ν , between the RFG model and the nominal LFG model. The resulting difference reaches up to about 10%, with the largest deviations observed in the Sub-GeV energy region.

CCQE normalizations. The default nuclear model in NEUT, the LFG model, is compared with an alternative model, the RFG model. Two independent uncertainties are implemented for the normalizations above and below 1330 MeV. For Sub-GeV ν_e and ν_μ CCQE events, the uncertainties are 5% and 1%, respectively. For Multi-GeV CCQE events, the uncertainties for ν_e and ν_μ flavors are 25%.

CCQE flavor ratios. These uncertainties also include comparisons between the LFG and SF models. The differences in the cross-section ratios, $(\bar{\nu}_e + \bar{\nu}_\mu)/(\nu_e + \nu_\mu)$ and $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$, between the two models are taken as the corresponding systematic uncertainties.

2p2h. A 100% normalization uncertainty is applied to the 2p2h process due to the lack of direct experimental measurements.

Single pion production uncertainties

Parameters: M_A^{RES} and C_A^5 . Independent uncertainties are applied to the primary input parameters of the Rein–Sehgal resonant pion production model. In each case, the input parameter is varied by its 1σ uncertainty, and the ratio of the double-differential cross section, $\frac{d^2\sigma}{dq^2 dW}$, in relative to the nominal model prediction, is taken as the corresponding 1σ effect. The uncertainties on the parameters are $M_A^{RES} = (0.95 \pm 0.15) \text{ GeV}/c^2$ and $C_A^5 = 1.01 \pm 0.12$.

Neutrino/antineutrino ratio. The uncertainty in the $\nu/\bar{\nu}$ single-meson production cross-section ratio is evaluated by comparing the default Rein–Sehgal model with an alternative model by Hernández *et al.* [140]. The uncertainty is calculated across four neutrino energy ranges, divided at 0.5, 1.5, and 3.0 GeV, and varies from 2.5% to 45%. All uncertainties are treated as fully correlated across energy ranges.

Charged/uncharged ratio. This uncertainty is derived by comparing the ratio of charged-pion ($\pi^\pm$) to neutral-pion (π^0) production between the Hernández model and the nominal Rein–Sehgal model. The 1σ variation corresponds to a 20% increase in the event weights for resonant π^0 production modes and a 20% decrease in the weights for resonant $\pi^\pm$ production modes.

Coherent pion production normalization. The uncertainty in the coherent pion production cross section is assigned to be 50% for NC and $CC\nu_e$ interactions, and 100% for the $CC\nu_\mu$ interaction.

DIS uncertainties

Below we summarize all crucial DIS uncertainties which are taken into account in this analysis.

DIS normalization. The uncertainty in the normalization of the DIS cross section is estimated from the difference between the NEUT prediction and the world-average measurements [26] for neutrino

energies between 30 GeV and 200 GeV. The resulting differences are 3.5% for neutrinos and 6.5% for antineutrinos.

PDF model difference. This uncertainty is defined as the difference in the DIS cross section obtained using the default GRV98 parton distribution function (PDF) [125] and an alternative PDF from the CKMT model [141]. The resulting difference ranges from 5% to 30% for neutrinos and from 10% to 50% for antineutrinos, with the largest deviations observed at low energies.

Vector and axial corrections on structure functions. Two corrections are applied to the structure functions $F_2(x, Q^2)$ and $F_3(x, Q^2)$, as described in Eq. (3.19) and [142]. The first correction accounts for higher-order QCD effects on the vector components of the structure functions. The second, referred to as the axial correction, allows the axial components to differ from the vector ones, which are otherwise assumed to be identical. The impacts of these two corrections are treated separately as systematic uncertainties.

Normalization in structure functions. Uncertainties in the normalization of the structure functions $F_2(x, Q^2)$ and $F_3(x, Q^2)$ result in a 3% uncertainty in the DIS cross section [142].

DIS hadron multiplicity. This uncertainty accounts for variations in the number of hadrons produced in DIS interactions for a given invariant hadronic mass, W . It is evaluated by computing the ratio of cross sections as a function of W between the GENIE [143] and NEUT event generators, scaled by the change in hadron multiplicity at each W value. The 1σ variation does not alter the total number of DIS events; instead, it increases or decreases the relative frequency of events with specific hadron multiplicities, before applying FSI effects.

Other uncertainties related to interactions

FSI and SI. For the FSI and SI models, the normalization parameters of the interaction channels, listed in Subsection 3.2.2 on page 49, are tuned to reproduce hadron–nucleus scattering data [128]. These parameters are then varied within their uncertainties, taking into account the correlations among them, and an event weight is computed for each parameter set. The largest and smallest resulting event weights are treated as the upper and lower bounds of the associated systematic uncertainty.

Neutron multiplicity. Two neutron-related systematic uncertainties are included in this analysis to account for the dependence on neutron production models in the neutron-based event selection used for SK–IV and SK–V data. The larger of the two arises from measurements of neutron production as a function of transverse momentum by T2K [144]. The second uncertainty accounts for differences in neutron multiplicities predicted by the NEUT and GENIE [143] neutrino interaction generators. Variations in these systematic uncertainties shift events between samples with no tagged neutrons and those with one or more tagged neutrons. The overall uncertainty in the number of events is estimated from these differences for the neutron-tagged samples, resulting in variations of up to 25% in the event weights for SK–IV–V.

ν_τ cross section. This uncertainty corresponds to a 25% variation in the rate of CC ν_τ interactions as a 1σ effect. The value is estimated from direct measurements of the ν_τ cross section. In 2018, SK published its own measurement with an uncertainty of 20% [145]. Incorporating this result—or the ν_τ identification techniques developed in that analysis—into future oscillation studies remains an important goal.

Reconstruction uncertainties

The uncertainties in event reconstruction, described in this section, are estimated and treated independently for each data-taking phase. Many detector-related uncertainties are evaluated using a “scale-and-shift” method. In this approach, for a reduction or sample classification defined by a cut on a variable x , the corresponding Monte Carlo variable, x_{MC} , is scaled and shifted according to

$$x'_{\text{MC}} = \alpha x_{\text{MC}} + \beta,$$

to best fit the data distribution, x_{data} . The difference in performance between the original x_{MC} and the fitted x'_{MC} is taken as the systematic uncertainty. In many cases, the statistical uncertainty from the fit is also added in quadrature to the total uncertainty.

Ring separation. The uncertainty in the separation between single- and multi-ring events is evaluated using the scale-and-shift method applied to the ring-counting likelihoods. The scaling and shifting parameters are independently varied for the single- and multi-ring components. After the fit, each parameter is varied within its fitted 1σ range, and the maximum variation in the number of single- and multi-ring events is taken as the systematic uncertainty. The uncertainty is evaluated separately for events within the conventional fiducial volume (FV) of the inner detector—defined as being more than 200 cm from the wall—and for events in the expanded FV region, located 100–200 cm from the wall, for each analysis sample.

Particle identification. The uncertainties in the separation between e-like and μ -like events are evaluated using the same scale-and-shift method applied to the particle identification (PID) parameter, as described for the ring-counting uncertainty. Two independent uncertainties are assigned for the single- and multi-ring samples. In the multi-ring case, the PID of the most energetic ring is used for the classification.

Two ring π^0 . The uncertainty in the event rate of the two-ring π^0 sample is evaluated using the same scale-and-shift method applied to the ring-counting and PID likelihoods, as described for the ring-separation and PID uncertainties. The scaling and shifting parameters are fitted separately for the first and second rings, and the resulting variations are combined to obtain a single overall uncertainty. As in previous cases, the uncertainty is evaluated independently for events within the conventional fiducial volume (FV) of the inner detector and for those in the expanded FV region.

The author of this monograph has contributed to the evaluation of systematic uncertainties related to ring separation, particle identification, and π^0 reconstruction in both the conventional and expanded FV regions.

Absolute energy calibration. This systematic uncertainty has been discussed in detail in Section 3.4 (page 54).

Dominant systematic uncertainties on the neutrino mass ordering

The systematic uncertainties that have the largest impact on the analysis sensitivity to the neutrino mass ordering are those which can modify the ratio of upward-going ($\cos\theta_z < 0$) to downward-going ($\cos\theta_z > 0$) events in the Multi-GeV e-like samples. Fig. 3.19 shows the dominant sources of systematic uncertainties that affect the ratio of upward- to downward-going events in the Multi-GeV e-like samples. Even after 24 years of atmospheric neutrino data taking with SK, the largest source of uncertainty remains statistical, indicating that the sensitivity to the neutrino mass ordering is still limited by

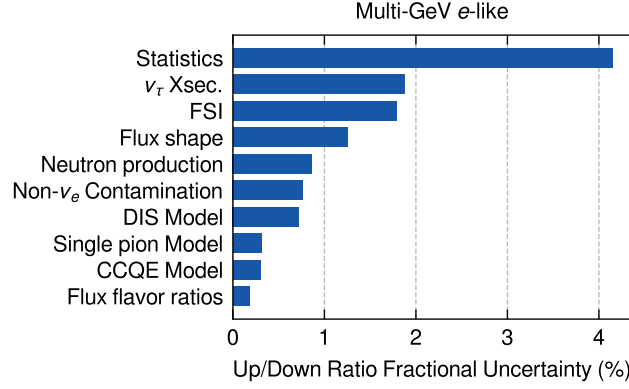


Figure 3.19. Estimated fractional uncertainties on the ratio of upward-going ($\cos\theta_z < 0$) to downward-going ($\cos\theta_z > 0$) events in the Multi-GeV e-like samples (both single- and multi-ring), incorporating contributions from both statistical and systematic uncertainties. Figure taken from [93].

statistics. The uncertainty in the ν_τ cross section is the second most significant contributor and plays a crucial role in the mass ordering analysis. Tau neutrino interactions observed at SK arise primarily from $\nu_\mu \rightarrow \nu_\tau$ oscillations, and therefore no downward-going ν_τ events exist to constrain the ν_τ cross-section uncertainty. Moreover, ν_τ interactions are typically reconstructed as Multi-GeV, multi-ring e-like events, placing them directly within the mass-ordering signal region [145]. In contrast, uncertainties related to the overall flux and cross-section normalizations have a smaller impact on the mass-ordering analysis, as their effects on the predicted event rates are largely independent of the zenith angle.

3.6.2. Atmospheric-only results

The results presented in this and the following sections are shown as $\Delta\chi^2$ contours of the oscillation parameters, calculated relative to the global best-fit value, with and without constraints on $\sin^2\theta_{13}$. When the reactor constraint is not applied, four neutrino oscillation parameters are simultaneously fitted in the analysis. These are: δ_{CP} , $\sin^2\theta_{13}$, $\Delta m_{32,31}^2$, and $\sin^2\theta_{23}$.

Fig. 3.20 shows the one-dimensional (1D) $\Delta\chi^2$ profiles of the fitted oscillation parameters, δ_{CP} , $\sin^2\theta_{13}$, $\Delta m_{32,31}^2$, and $\sin^2\theta_{23}$, from the atmospheric-only analysis. The contours are shown relative to the global best-fit point across both mass orderings, with blue and orange lines representing the normal (NO) and inverted (IO) orderings, respectively. As can be seen, the normal ordering is preferred. The difference between the minimum χ^2 values for the two orderings is $\Delta\text{MO} = 5.23$. The MC expectation for the mass-ordering sensitivity at the best-fit oscillation parameters is $\Delta\text{MO} = 1.53$, indicated by the dashed lines in Fig. 3.20.

Sensitivity to δ_{CP} in the SK data arises from both the Sub-GeV and Multi-GeV e-like samples. For δ_{CP} values near $-\pi/2$, an enhancement in the ν_e appearance rate and a corresponding reduction in the $\bar{\nu}_e$ appearance rate are expected relative to the case of $\delta_{\text{CP}} = 0$. The best-fit value of δ_{CP} is -1.89 for both mass orderings, consistent with the observed increase in ν_e appearance.

The preferred values of $\sin^2\theta_{13}$ are 0.020 and 0.010 for the normal and inverted orderings, respectively. Both results are consistent with the reactor-preferred value of $\sin^2\theta_{13} = 0.0220$ at the 90% confidence level, while $\sin^2\theta_{13} = 0$ is disfavored in the normal ordering at the 99% confidence level.

3.6.3. Results with reactor constraints

The most precise measurements of $\sin^2\theta_{13}$ currently come from the Daya Bay, RENO, and Double Chooz experiments (as discussed in Subsection 1.3.3, page 21). In the analysis described below, the

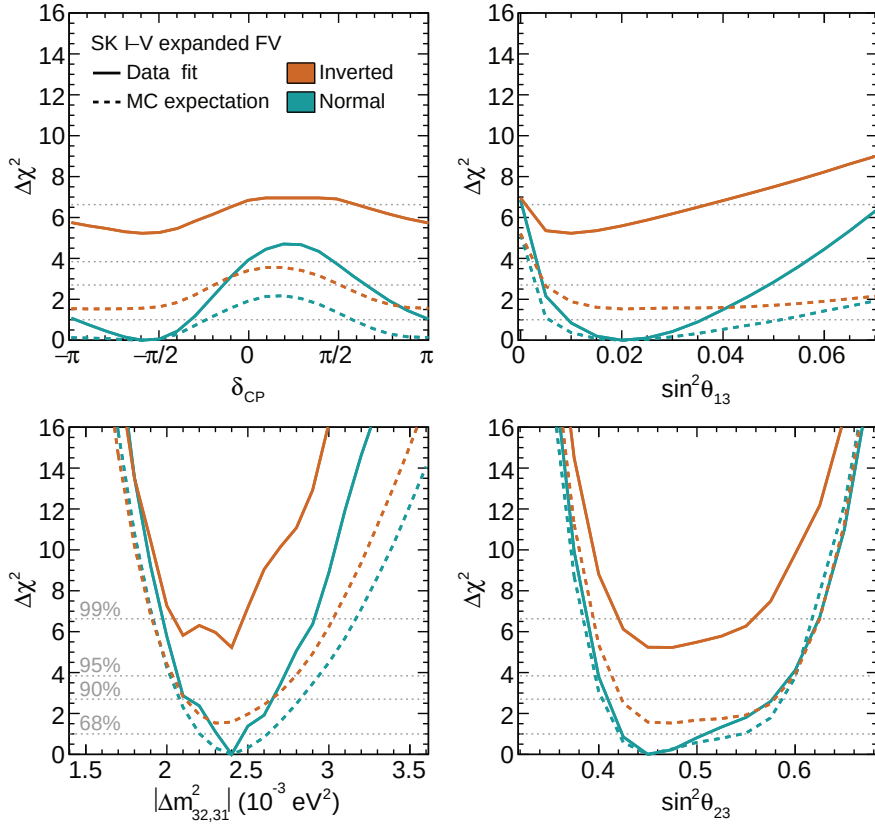


Figure 3.20. One-dimensional $\Delta\chi^2$ profiles of the oscillation parameters in the analysis with $\sin^2\theta_{13}$ treated as a free parameter. Solid lines indicate the fit to data, while dashed lines show the MC expectation at the data best-fit oscillation parameters (see Table 3.3). Horizontal dashed lines represent the critical values of the χ^2 distribution for one degree of freedom, corresponding (from lowest to highest) to confidence levels of 68%, 90%, 95%, and 99%. Figure taken from [93].

central value of this parameter is taken to be $\sin^2\theta_{13} = 0.0220 \pm 0.0007$, based on the average of these measurements reported in [26]. Fig. 3.21 shows the 1D $\Delta\chi^2$ profiles for the fitted neutrino oscillation parameters.

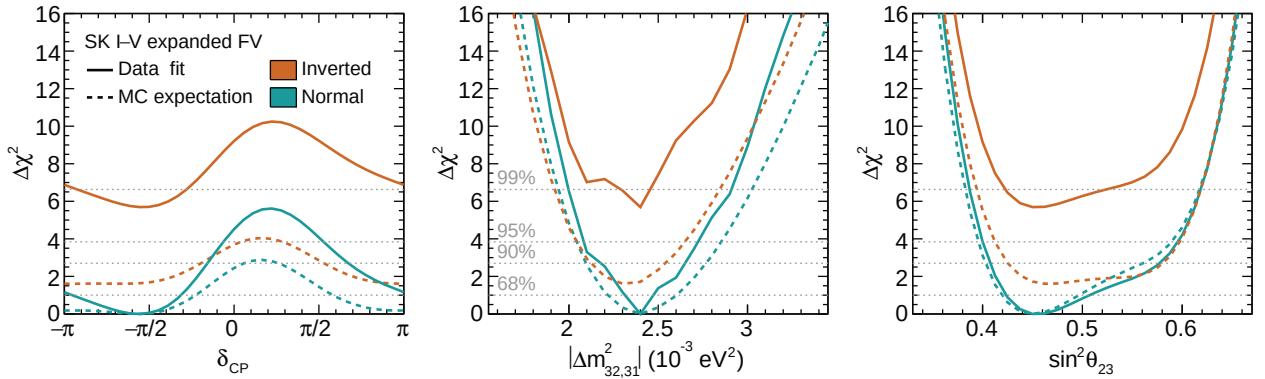


Figure 3.21. One-dimensional (1D) $\Delta\chi^2$ profiles of the oscillation parameters in the analysis with $\sin^2\theta_{13}$ constrained. Solid lines indicate the fit to data, while dashed lines show the MC expectation at the data best-fit oscillation parameters (see Table 3.3). Horizontal dashed lines represent the critical values of the χ^2 distribution for one degree of freedom, corresponding to confidence levels of 68%, 90%, 95%, and 99%. Figure taken from [93].

The best-fit value of δ_{CP} in both the normal and inverted mass orderings for the fit with $\sin^2\theta_{13}$ constrained is -1.75 , consistent with the atmospheric-only result at the 1σ level.

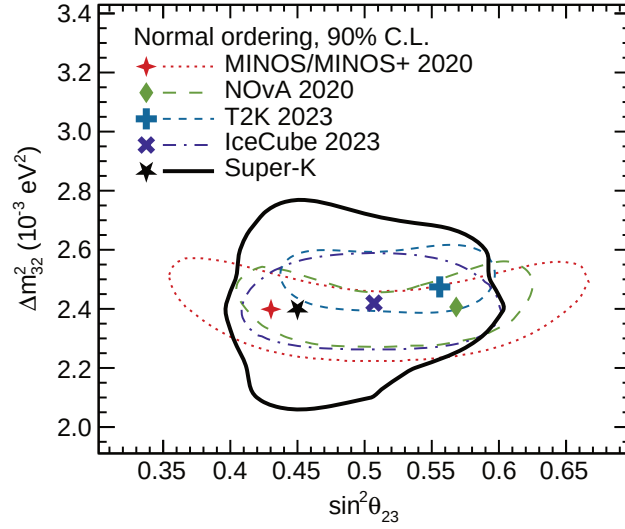


Figure 3.22. Two-dimensional $\Delta\chi^2$ contours of the oscillation parameters Δm_{32}^2 and $\sin^2 \theta_{23}$ for the normal mass ordering. The contours correspond to the 90% confidence level, assuming two degrees of freedom, with $\Delta\chi^2$ values computed relative to the best-fit point for each experiment in the normal ordering. The SK contour represents the result of this analysis, while the other contours are adapted from the published results of MINOS+ [146], NOvA [147], T2K [46], and IceCube [148]. Best-fit points for each experiment are indicated by the corresponding markers. Figure taken from [93].

Fig. 3.22 presents the constraints on $\sin^2 \theta_{23}$ and Δm_{32}^2 obtained from the $\sin^2 \theta_{13}$ -constrained analysis of the SK atmospheric neutrino data, together with results from MINOS/MINOS+ [146], NOvA [147], T2K [46], and IceCube [148]. The SK atmospheric results are in good agreement with those from the other long-baseline and atmospheric experiments at the 90% confidence level. The current analysis favors a best-fit value of $\sin^2 \theta_{23}$ in the lower octant, whereas the previous SK publication [94] preferred a value in the upper octant. Nevertheless, the values of $\sin^2 \theta_{23}$ in both octants remain compatible with the data within the 1σ confidence interval.

Table 3.4. Best-fit neutrino oscillation parameters from the analyses presented in this work. The uncertainties on each oscillation parameter are the $\pm 1\sigma$ allowed regions assuming a χ^2 distribution with one degree of freedom. The second-to-last column shows the total χ^2 . Both analyses have 930 bins. Taken from [93].

Fit result	Ordering	$ \Delta m_{32,31}^2 $ (10^{-3} eV^2)	$\sin^2 \theta_{23}$	$\sin^2 \theta_{13}$	δ_{CP} ($-\pi, \pi$)	χ^2	$\Delta\chi_{\text{I.O.}-\text{N.O.}}^2$
SK, Atmospheric Only	Normal	$2.40^{+0.07}_{-0.09}$	$0.45^{+0.06}_{-0.03}$	$0.020^{+0.016}_{-0.011}$	$-1.89^{+0.87}_{-1.18}$	1022.06	5.23
	Inverted	$2.40^{+0.05}_{-0.33}$	$0.48^{+0.07}_{-0.05}$	$0.010^{+0.021}_{-0.008}$	$-1.89^{+1.32}_{-1.97}$	1027.29	
SK, $\sin^2 \theta_{13}$ Constrained	Normal	$2.40^{+0.07}_{-0.09}$	$0.45^{+0.06}_{-0.03}$	—	$-1.75^{+0.76}_{-1.25}$	1022.06	5.69
	Inverted	$2.40^{+0.06}_{-0.12}$	$0.45^{+0.08}_{-0.03}$	—	$-1.75^{+0.89}_{-1.22}$	1027.75	

3.6.4. Interpretation

It is well known that the significance of a neutrino mass-ordering determination does not necessarily correspond to the difference between the minimum χ^2 values obtained under the two ordering hypotheses [149]. This is because the normal and inverted orderings do not constitute nested hypotheses, and therefore Wilks' theorem [150] does not apply. To properly evaluate the ordering significance in this analysis, ensembles of pseudo-data sets are generated using the atmospheric neutrino MC

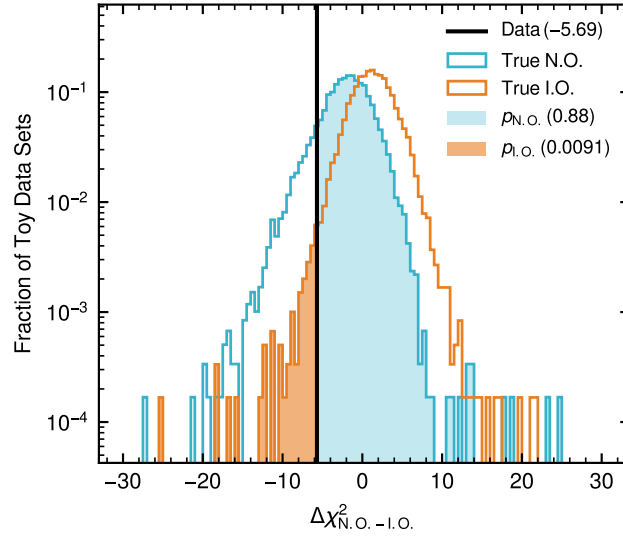


Figure 3.23. Distribution of the mass-ordering test statistic, $\Delta\chi^2$, obtained from ensembles of simulated pseudo-data sets assuming either the normal or inverted mass ordering. The result from the atmospheric analysis with $\sin^2 \theta_{13}$ constrained is shown by the vertical black line. The blue and orange histograms represent the distributions of this statistic for pseudo-data sets generated under the normal and inverted ordering hypotheses, respectively. Figure taken from [93].

simulation. These ensembles are used to estimate the p -value for obtaining a $\Delta\chi^2$ difference between the mass-ordering hypotheses that is equal to or more extreme than that observed in the data.

Fig. 3.23 presents the distribution of $\Delta\chi^2$ values compared with the data fit result from the atmospheric analysis with $\sin^2 \theta_{13}$ constrained. The probability of observing a result more extreme than that obtained from the data (the p -value) corresponds to the area under the distribution to the right of the data line for the normal ordering hypothesis, and to the left of the data line for the inverted ordering hypothesis. In this study, the p -value derived from ensembles of simulated pseudo-data sets for the inverted mass ordering is 0.0091. With the current SK exposure, however, the statistical power remains insufficient to decisively exclude either mass ordering. The p -value associated with the data under the normal ordering hypothesis, 0.88, is likewise not particularly high, reflecting the limited discriminating sensitivity. In scenarios where two mutually exclusive hypotheses must be tested simultaneously, the CL_s method [151] provides a modified statistical approach that accounts for the level of compatibility of the data with both hypotheses.

$$\text{CL}_s = \frac{p_{\text{I.O.}}}{1 - p_{\text{N.O.}}}, \quad (3.31)$$

where $p_{\text{N.O.}}$ and $p_{\text{I.O.}}$ refer to the p -values in the normal or inverted ordering. The CL_s for the atmospheric analysis with reactor constrained is 0.077, corresponding to a rejection of the inverted mass ordering at the 92.3% confidence level. This number is very similar to the previous SK result $\text{CL}_s = 0.070$ [94].

In this chapter, we focus on the first results presented by the Super-Kamiokande and T2K Collaborations concerning the joint measurement of neutrino oscillation parameters using both atmospheric and beam neutrino data [152]. This analysis is based on 3244.4 days of Super-Kamiokande atmospheric neutrino data collected during the SK-IV period (2008–2018), corresponding to approximately 50% of the statistics used in the analysis described in the previous chapter. Details of the atmospheric neutrino oscillation analysis are provided in [153]. For the T2K beam neutrinos, an exposure of $19.7 (16.3) \times 10^{20}$ protons on target in neutrino (antineutrino) mode is used [46].

4.1. Motivation for the joint neutrino oscillation analysis

Both the T2K and Super-Kamiokande (SK) experiments employ the SK detector for their neutrino oscillation measurements. As discussed in Section 2.2 on page 33, T2K's off-axis neutrino beam produces a narrow-band energy spectrum peaked at $E_\nu \simeq 600$ MeV, with a well-defined direction for beam-induced events at SK. This configuration enables a precise measurement of the ν_μ *disappearance* probability, which appears as a characteristic dip in the observed ν_μ energy spectrum near $E_\nu \simeq 600$ MeV. From this *disappearance* pattern, T2K constrains the oscillation parameters Δm_{32}^2 and $\sin^2 \theta_{23}$, which correspond to the position and amplitude of the δ_{CP} , respectively. Both the neutrino mass ordering and the value of $\sin \delta_{\text{CP}}$ influence the *appearance* probabilities for neutrinos and antineutrinos, $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$, in an asymmetric manner. A major strength of the T2K experiment lies in its ability to operate in both neutrino and antineutrino beam modes independently. This capability allows a direct comparison between ν and $\bar{\nu}$ oscillations, providing sensitivity to δ_{CP} and partial discrimination between the normal and inverted mass orderings based on the relative numbers of ν_e and $\bar{\nu}_e$ events observed at the far detector. On the other hand, the similarity between the effects of the neutrino mass ordering and the value of δ_{CP} can reduce the experiment's ability to independently determine these parameters, a limitation that affects T2K in particular. An example of this degeneracy is illustrated in Figure 4.1, where the numbers of observed electron neutrino and antineutrino events are plotted against each other [46]. Overlaps in the predicted event yields of ν_e and $\bar{\nu}_e$ appear for different values of δ_{CP} under the normal and inverted ordering hypotheses. Consequently, the determination of δ_{CP} becomes challenging when the neutrino mass ordering is not known.

For SK atmospheric neutrino events, the constraints on Δm_{32}^2 and $\sin^2 \theta_{23}$ are comparatively weaker (as illustrated, for example, in Fig. 3.22 on page 75) due to the limited knowledge of the incoming neutrino direction and the broad range of neutrino energies involved. However, upward-going

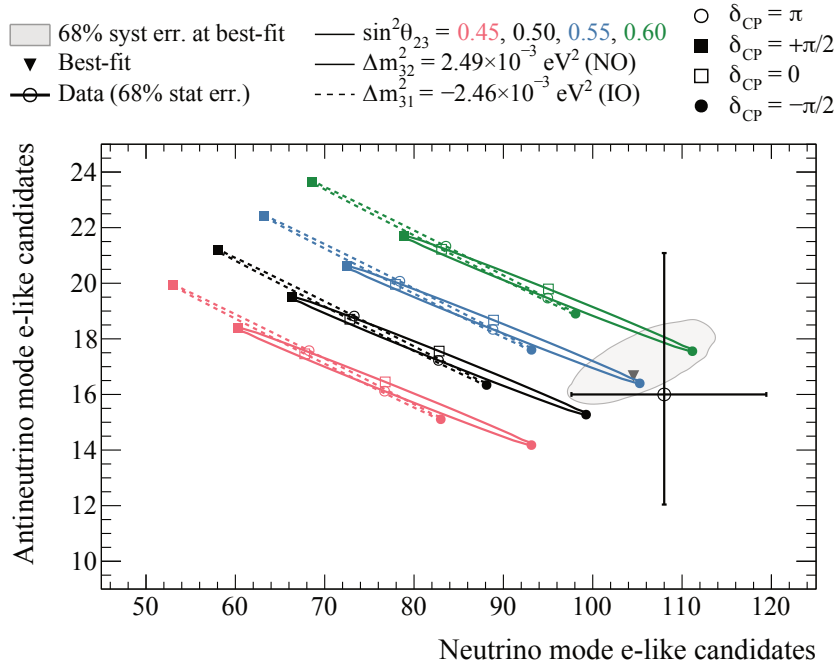


Figure 4.1. Number of e-like neutrino events versus e-like antineutrino events, with predicted event yields for ν_e shown for different sets of oscillation parameters as colored ellipses. The figure is taken from [46].

neutrinos traverse long paths through the Earth and experience significant matter effects, which asymmetrically alter the oscillation probabilities of neutrinos and antineutrinos depending on the mass ordering (see Section 3.1 on page 45 for details). In particular, (anti)neutrinos with energies between 2 and 10 GeV—commonly referred to as the Multi-GeV range—experience the strongest resonant matter effects when traversing the Earth’s core, assuming the normal (inverted) mass ordering. The magnitude of this resonance is proportional to $\sin^2 \theta_{23}$, providing sensitivity to both the θ_{23} octant and the neutrino mass ordering through an excess of upward-going ν_e or $\bar{\nu}_e$ events in this energy region. In a combined analysis, these atmospheric neutrino samples help break the degeneracy between mass ordering and δ_{CP} that affects T2K alone. Furthermore, the joint fit also benefits from the mass-ordering sensitivity of T2K’s lower-energy ν_e appearance data, providing complementary information across different energy regimes.

4.2. Neutrino event reconstruction with FITQUN

Since both the SK atmospheric neutrinos and T2K accelerator neutrinos are observed in the same detector, a common event reconstruction algorithm is employed in the joint analysis. This algorithm is based on a likelihood approach and represents an advancement over the reconstruction method APFIT (details of this reconstruction algorithm are given in the Subsection 3.3 on page 54). The new reconstruction software, FITQUN, was originally developed for the MiniBooNE experiment [154] and later adopted for the results presented by the SK experiment in [153] and by T2K in [46]. The FITQUN algorithm employs a maximum-likelihood method to simultaneously reconstruct particle types and determine their kinematic properties within the detector. Compared to the previous fitting algorithm, APFIT, FITQUN incorporates additional information such as PMT hits outside the Cherenkov cone and detailed hit timing, thereby improving reconstruction accuracy.

The fitting process in FITQUN is executed multiple times to determine the optimal kinematic parameters (e.g., vertex position, momentum, and direction) for each possible particle hypothesis,

ultimately selecting the configuration with the highest likelihood. In contrast, APFIT performs a single fit, determining the kinematic parameters and particle type sequentially. Since the launch of SK in 1996, the substantial increase in computing power has enabled FITQUN to achieve significantly improved reconstruction accuracy while maintaining a computational cost comparable to that of APFIT.

4.2.1. Fitting procedure

The FITQUN reconstruction procedure can be divided into four main stages. In the first stage, the interaction vertex is roughly estimated using PMT timing information. The second stage, referred to as *hit clustering*, groups PMT hits in time to identify candidates for particle activity. The third stage performs single-ring reconstruction, in which events are fitted under the assumption that only one light-producing particle is present. Three particle hypotheses are considered in this step: electron, muon, and charged pion. For each particle hypothesis, the kinematic parameters of the event—including the interaction vertex, particle momentum, and direction—are varied to maximize the likelihood function with respect to the observed data. Particle identification (PID) is then determined from the best-fit likelihood values among the hypotheses. For example, electrons and muons are distinguished by applying a selection on $\ln(L_e/L_\mu)$, the logarithm of the likelihood ratio between the best-fit electron and muon hypotheses.

The distribution of the PID variable is shown in Fig. 4.2 for the FC atmospheric neutrino data and corresponding MC simulation results. Details of the FC event selection are provided in Section 3.5 on page 57. The distributions are presented separately for Sub-GeV ($E_{\text{vis}} < 1330$ MeV) and Multi-GeV ($E_{\text{vis}} > 1330$ MeV) samples. In both energy regions, a clear separation of the PID likelihood variable is observed between electron-like (e-like) and muon-like (μ -like) events.

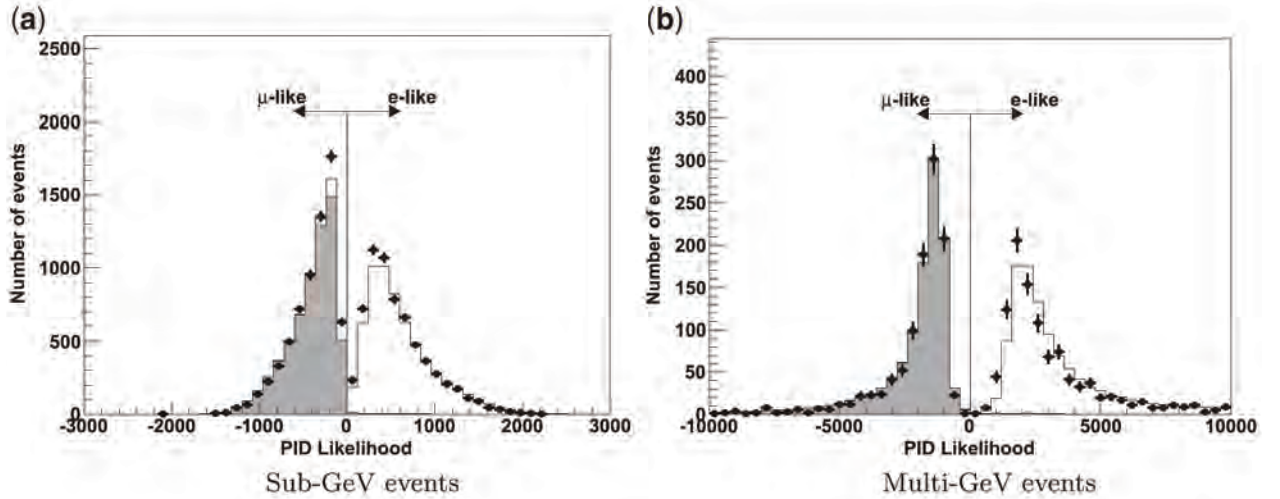


Figure 4.2. PID likelihood distributions for fully contained single-ring events reconstructed with FITQUN. The (*left panel*) shows the Sub-GeV sample, and the (*right panel*) shows the Multi-GeV sample. Black points represent the data, while the grey histograms indicate the atmospheric neutrino MC prediction after oscillations. The MC expectations are generated assuming the normal neutrino mass ordering, with oscillation parameters equal to $\sin^2 \theta_{13} = 0.0210$, $\sin^2 \theta_{23} = 0.5$, $\Delta m_{32}^2 = 2.4 \times 10^{-3} \text{ MeV}/c^2$, and $\delta_{\text{CP}} = 0$. The figure is taken from [153].

The distribution of the distance between the reconstructed and true vertices for FC single-ring CCQE events in the atmospheric neutrino MC is shown in Fig. 4.3. Only events with true interaction vertices located within the conventional fiducial volume are included. The vertex resolution is evaluated as the distance corresponding to the 68% of these distributions. A clear improvement in vertex reconstruction performance is observed with FITQUN compared to APFIT.

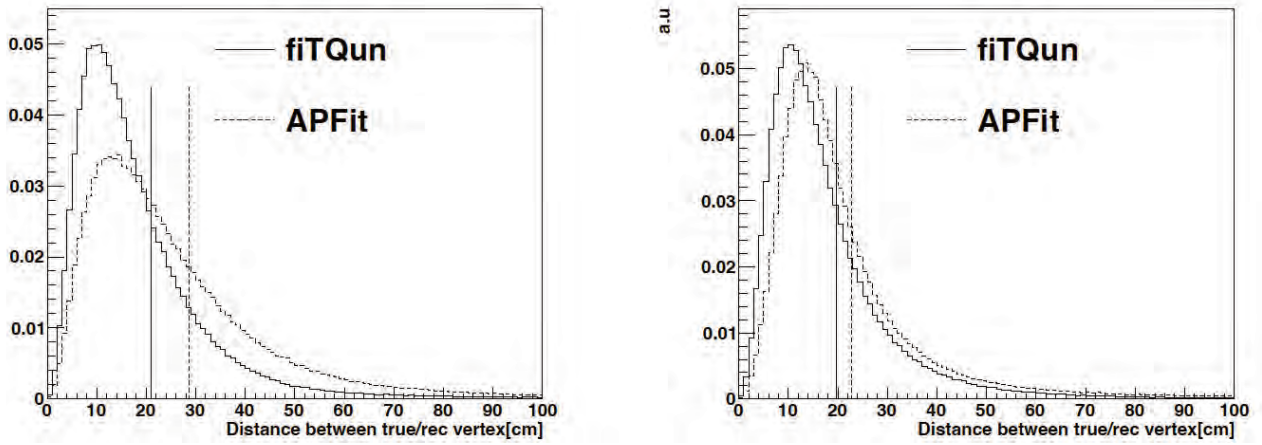


Figure 4.3. Distribution of the distance between the reconstructed and true vertices for FC single-ring CCQE events from the atmospheric neutrino MC. Charged-current ν_e events are shown in the (*left panel*), and charged-current ν_μ events in the (*right panel*). Results are presented for two reconstruction algorithms: APFIT (dashed line) and FITQUN (solid line). The figure is taken from [153].

Fig. 4.4 shows the vertex resolution as a function of visible energy. For CCQE ν_e events, the vertex resolution obtained with FITQUN remains stable at approximately 20.6 cm across the visible energy range from 100 MeV to 1330 MeV, while the resolution from APFIT is significantly worse, varying from 34.6 cm to 25.3 cm over the same range. Similarly, for CCQE ν_μ events, FITQUN achieves a resolution ranging from 29.2 cm to 15.9 cm, outperforming APFIT, whose resolution varies between 34.2 cm and 18.2 cm.

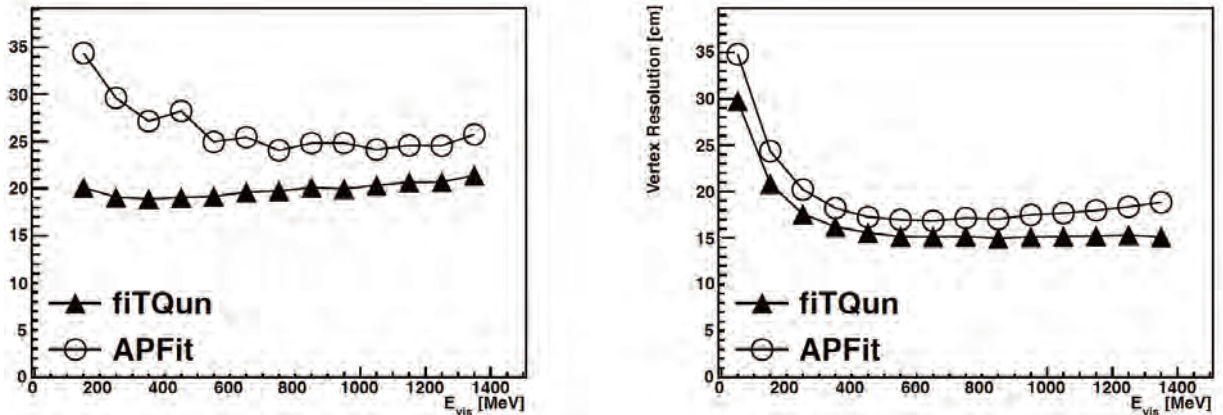


Figure 4.4. Vertex resolution of FC CCQE events as a function of visible energy. The (*left panel*) shows results for CC ν_e events, and the (*right panel*) for CC ν_μ events. Results obtained with the FITQUN reconstruction are indicated by filled triangles, while those from APFIT are shown with open circles. The figure is taken from [153].

The performance of FITQUN for direction reconstruction was verified to be consistent with that reported in [155]. The distribution of the ratio between reconstructed and true momenta was also examined in that study. In the same visible energy range, the momentum resolution of FITQUN for CCQE ν_e events improves from 5.39% to 2.58% as the visible energy increases, while the resolution of APFIT varies from 7.04% to 3.32% [155]. Furthermore, FITQUN demonstrates an enhanced ability to discriminate between electrons and muons, achieving a misidentification rate below 1% in the same energy range. A summary of the reconstruction performance for both algorithms—FITQUN and APFIT—is presented in Table 4.1, based on FC single-ring CCQE event samples for both ν_e and ν_μ .

Table 4.1. Basic performance comparison of the reconstruction algorithms FITQUN (new) and APFIT (old), evaluated using fully contained (FC) single-ring CCQE event samples for ν_e and ν_μ . Taken from [153].

Reconstruction	FITQUN	APFIT
True CCQE ν_e sample		
Vertex Resolution	20.6 cm	24.9 cm
Direction Resolution	1.48°	1.68°
Momentum Resolution	2.90%	3.56%
Mis-PID rate	0.02%	0.50%
True CCQE ν_μ sample		
Vertex Resolution	15.8 cm	17.3 cm
Direction Resolution	1.00°	1.28°
Momentum Resolution	2.26%	2.60%
Mis-PID rate	0.05%	0.91%

The final and most crucial step in the FITQUN reconstruction process concerns multi-ring reconstruction, in which events are fitted under hypotheses involving multiple final-state particles. As discussed previously, a large fraction of atmospheric neutrino interactions at Multi-GeV energies produce multi-ring topologies. These events are particularly important for achieving high sensitivity in neutrino mass ordering studies (see Section 3.1 on page 45).

Accurate reconstruction of events containing multiple light-producing particles is essential for the atmospheric neutrino oscillation analysis. In FITQUN, the multi-ring fitting procedure is applied only to the first time window, corresponding to the primary event trigger, and not to any delayed windows, to reduce computational cost. The multi-ring hypothesis is constructed iteratively by adding an additional ring to the result of the previous fit. Each of the three particle-type hypotheses used in the single-ring reconstruction—electron-like, μ -like, and π^+ -like—is tested for the newly added ring. A new ring is accepted only if the likelihood ratio between the updated fit and the original fit satisfies the predefined criterion for ring counting.

The likelihood ratios between the 2-ring and 1-ring hypotheses for both Sub-GeV and Multi-GeV events are shown in Fig. 4.5.

The performance of the ring counting in both APFIT and FITQUN using atmospheric neutrino MC events is summarized also in Table 4.2. In this table, only charged particles with energies exceeding 30 MeV above the Cherenkov threshold in the final state—making them observable—are considered as true ring candidates. No requirement is imposed on their angular separation from other particles. FITQUN demonstrates a higher capability for reconstructing multi-ring events compared to APFIT, while both algorithms achieve comparable performance in correctly identifying single-ring topologies. Although the fake-ring fraction of FITQUN is slightly larger than that of APFIT for true single- and two-ring event reconstruction, the increase is not sufficient to offset the improvement from the higher fraction of correctly reconstructed events.

The PID likelihood distribution of the most energetic ring in fully contained multi-ring events is shown in Fig. 4.6. The separation between electron-like and μ -like rings is not as distinct as in the single-ring events shown in Fig. 4.2, primarily because some Cherenkov rings in multi-ring events overlap. Nevertheless, clear peaks corresponding to both electron-like and μ -like events are still visible.

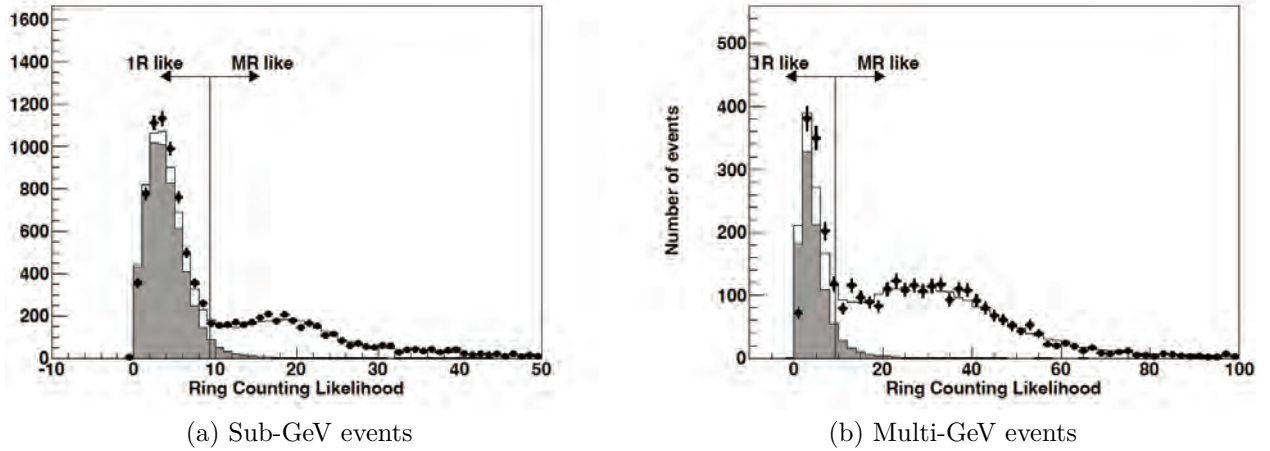


Figure 4.5. Distribution of the ring-counting likelihood ratio between the best-fit single-ring and multi-ring hypotheses for FC atmospheric neutrino events. The most energetic ring is required to be electron-like. The (*left panel*) shows the result for Sub-GeV events, while the (*right panel*) shows the result for Multi-GeV events. Atmospheric neutrino data are shown as points, and the MC prediction including neutrino oscillations is shown as the histogram. The figure is taken from [153].

Table 4.2. The basic performance of reconstruction algorithms of the ring counting: new FITQUN and old APFIT are presented. Taken from [155].

True number of rings	FITQUN			APFIT		
	1 ring	2 ring	≥ 3 rings	1 ring	2 ring	≥ 3 rings
True 1 ring	95%	4.64%	0.41%	95.9%	3.85%	0.29%
True 2 rings	27.8%	66.7%	5.56%	42.5%	52.8%	4.63%
True ≥ 3 rings	7.04%	25.5%	67.5%	20.2%	33.0%	46.8%

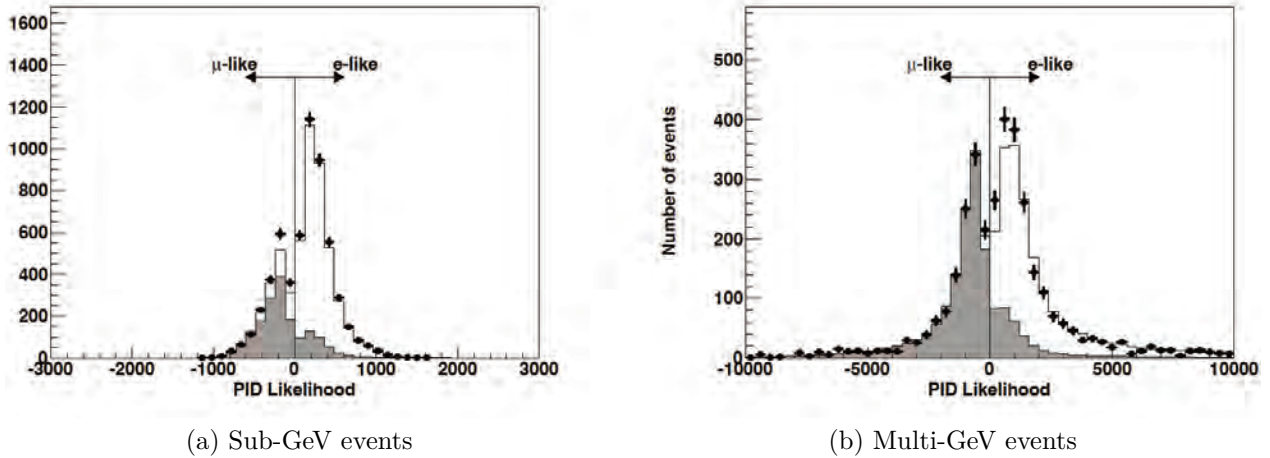


Figure 4.6. Distribution of the PID likelihood for the most energetic ring in FC multi-ring events. The (*left panel*) shows the result for Sub-GeV events, while the (*right panel*) shows the result for Multi-GeV events. Atmospheric neutrino data are shown as points, and the MC prediction including neutrino oscillations is shown as the histogram. The figure is taken from [153].

FiTqun usage for SK-I-III phases. As mentioned in Section 2.1.1 on page 26, SK has undergone eight distinct detector phases to date, and only the data from the longest-running SK-IV phase were used for the first joint SK-T2K oscillation analysis. At the time of publication [153], the data collected

during the SK-I–III phases were excluded due to significant data–MC discrepancies observed in the reconstructed variable distributions from FITQUN. Further improvements in the detector simulation are required before those data can be reliably included. Compared to APFIT, the FITQUN algorithm incorporates additional information—such as PMT hits outside the Cherenkov cone and detailed timing information—allowing a more precise fit. Consequently, discrepancies between data and MC that were previously negligible in less-precise reconstructions can become more pronounced when using FITQUN. Ongoing work focuses on validating the SK-I–III data and improving their agreement with simulation, with the goal of incorporating them into future iterations of the joint SK–T2K oscillation analysis.

4.3. Neutrino event selections

As discussed in detail in Section 3.5 on page 57, the SK detector classifies neutrino interactions as fully contained (FC), partially contained (PC), or upward-going muon (Up- μ) events. All three categories are used in the joint analysis discussed here. From the T2K beam neutrino analysis, however, only FC events are utilized for the joint neutrino oscillation study.

In the following subsections, we will describe the event selections for both atmospheric and T2K beam neutrinos. Given that the atmospheric neutrino selections were already discussed in detail in Chapter 3, we will focus here on highlighting the differences between the selections used in this SK–IV analysis—based on the FITQUN reconstruction—and those employed in the SK-I–III atmospheric neutrino analyses, which were based on the APFIT reconstruction.

4.3.1. Fiducial volume expansion with FITQUN

The fiducial volume (FV) of the SK detector was previously described in Section 3.4 on page 54. As a reminder, the *conventional* FV is defined as the region of the detector in which the reconstructed neutrino interaction vertex lies at least 200 cm from the nearest SK wall. This definition applies to both reconstruction algorithms, FITQUN and APFIT. As discussed in that section, an *expanded* FV was introduced in earlier analyses using APFIT, allowing vertices to be as close as 100 cm to the nearest wall. With the improved reconstruction performance of FITQUN, it became possible to extend the FV even further, increasing the usable detector volume beyond what was achievable with APFIT. It is important to note that in both reconstruction schemes, the FV expansion is applied only to fully contained (FC) events—thus including all T2K beam neutrino events and all SK FC subsamples from SK–IV phase.

However, it is important to note that enlarging the FV not only increases statistics but also introduces a risk of reduced signal purity in the selected neutrino samples. The reconstruction performance may degrade for events occurring close to the inner detector (ID) wall, as particles traveling toward the wall can produce only a few PMTs hits, resulting in poorly imaged Cherenkov rings. Moreover, as the FV boundary approaches the wall, the likelihood of interactions occurring outside the ID but being reconstructed inside the FV increases. As shown in previous sections, this effect arises from the finite vertex resolution, which for single-ring events is on the order of 20 cm. To assess the feasibility of expanding the FV, the distance to the nearest wall was studied for both signal and background components relevant to each sample, using reconstructed variables from FITQUN.

In Fig. 4.7, the fraction of events in the FC Sub-GeV μ -like 0-decay-e sample is shown as a function of the distance between the reconstructed vertex (from FITQUN) and the nearest inner detector wall (D_{wall}). Up to about 50 cm, the purity of signal events—defined as the fraction of $\nu_\mu\text{CC}$ and $\bar{\nu}_\mu\text{CC}$ interactions (shown in red)—remains stable. A similar behavior is observed for all FC SK event

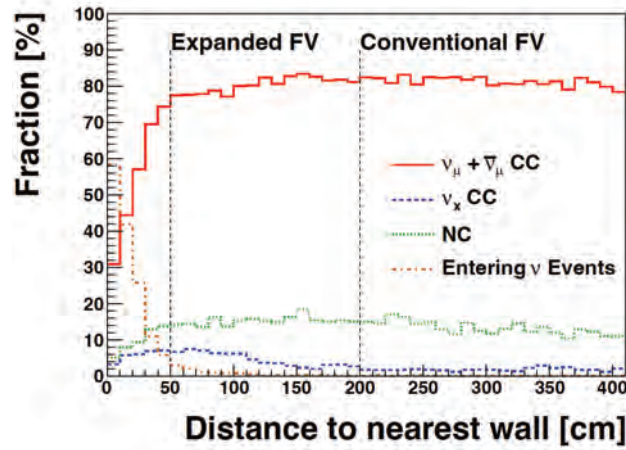


Figure 4.7. The fractional contributions of each component in the FC Sub-GeV μ -like 0-decay-e event sample from the atmospheric neutrino MC simulation, shown as a function of the reconstructed distance from the interaction vertex to the nearest inner detector wall. The figure is taken from [153].

categories, indicating that the FV cannot be expanded into the region with $D_{wall} < 50$ cm, where the fraction of entering neutrino events increases significantly.

In addition to the D_{wall} cut, the FV expansion also makes use of a new variable, T_{owall} , defined as the distance from the reconstructed vertex to the detector wall along the direction of the outgoing particle. This variable allows the inclusion of events that are geometrically close to the wall (small D_{wall}) but directed away from it (large T_{owall}), which typically produce enough PMTs hits to form a well-defined Cherenkov ring. This effect is demonstrated in Fig. 4.8.

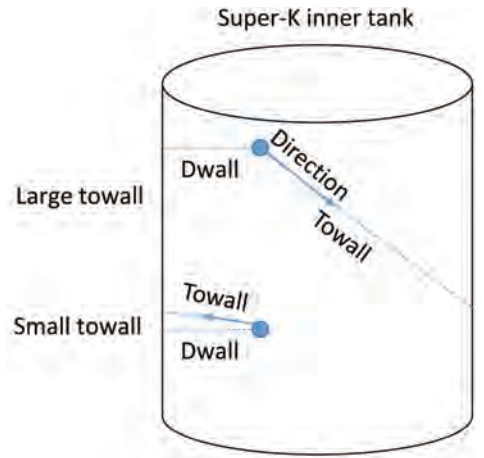


Figure 4.8. Definition of the D_{wall} and T_{owall} variables reconstructed with FITQUN. The figure is taken from [155].

A combination of D_{wall} and T_{owall} selections was used for all T2K beam neutrino events, which will be discussed further.

4.3.2. SK atmospheric samples

The SK atmospheric data set consists of 13 subsamples of FC events. The selection criteria are largely identical to those described in Section 3.5 on page 57, except that this analysis does not use neutron-tagging information on hydrogen. The overall scheme of the FC event classification is illustrated in Fig. 4.9. In this analysis, the number of decay electrons is used to help distinguish neutrino events from antineutrino events. Additionally, for the multi-ring samples, the boosted decision tree (BDT) classification used in more recent analyses has been replaced by the previous likelihood-based method [138].

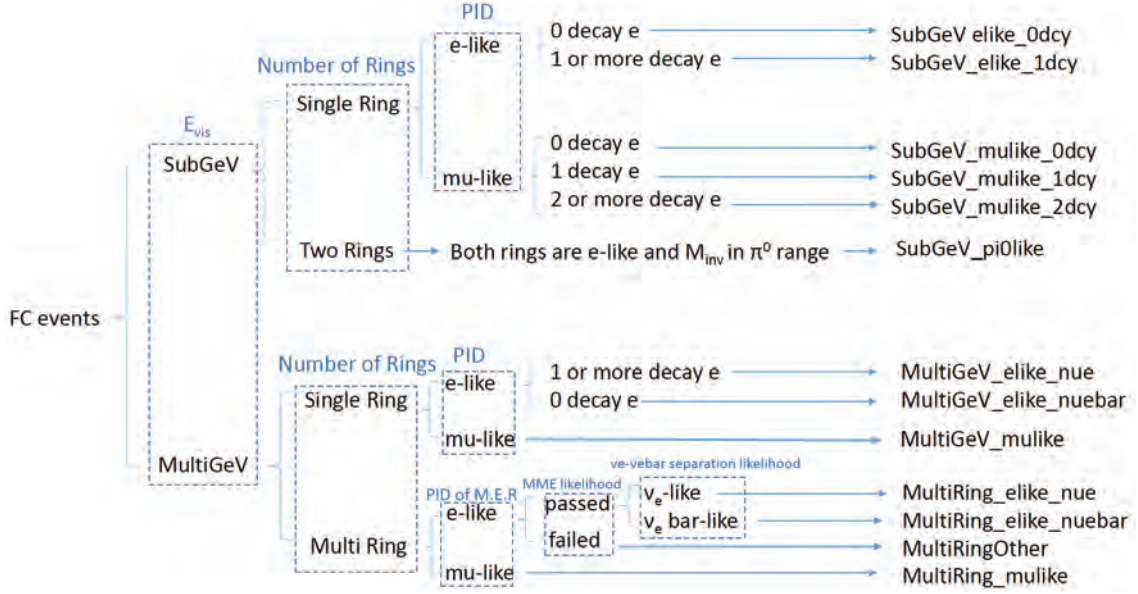


Figure 4.9. Scheme of the FC event selections used for this analysis of the atmospheric neutrinos for SK-IV data. The figure is taken from [155].

The selections for the PC and Up- μ samples are the same as described in Section 3.5 on page 57.

4.3.3. T2K beam neutrino samples at the far detector

As previously introduced in Section 2.2 on page 33, the T2K experiment performs its neutrino oscillation analyses using measurements from both the near detector (ND280) and the far detector (SK). In the following, we focus only on the neutrino selections applied at SK, as these are directly relevant to the work presented here. For readers interested in the neutrino selections at ND280, a detailed description can be found in [46]. As discussed in Subsection 2.3.2 on page 40, the T2K beam has an average neutrino energy of approximately 600 MeV. Since the charged-current (CC) ν_τ interactions require a neutrino energy threshold of about 3.5 GeV, they are kinematically forbidden or strongly suppressed in this energy range, and thus do not contribute significantly to this analysis.

At the T2K far detector, five signal-enriched samples are used, each containing a single reconstructed Cherenkov ring. Samples with a single reconstructed μ -like ring (1R μ) and a single reconstructed e-like ring (1Re) target ν_μ and ν_e charged-current quasi-elastic (CCQE) interactions in both forward horn current (FHC) and reverse horn current (RHC) beam modes (see Section 2.2.1 on page 34). An additional e-like sample, used only in the FHC data, targets CC1 π^+ interactions in which the π^+ is below the Cherenkov threshold. These π^+ are produced mostly from Δ^{++} and Δ^+ decays, as discussed in Section 3.2.2, with average energy of the ν_e above 1 GeV as Δ mass is equal to 1232 MeV/ c^2 . Such events are identified by the presence of a delayed μ -decay electron following the prompt electron from the CC interaction, forming the (1Re + 1d.e.) sample. The CCQE-like selection criteria are identical for both the FHC and RHC samples. As described previously, all selected events are required to be FC and to exhibit a single Cherenkov ring.

In general, in T2K, event classification into signal and background samples is performed using selection cuts based on the best-fit likelihood ratios between competing hypotheses, defined as

$$\Lambda_\alpha^\beta = \log \frac{L_\alpha}{L_\beta},$$

where α and β represent the respective signal-like and background-like hypotheses. In that case

classification into e-like or μ -like categories is performed using the likelihood ratio between the best-fit e-like and μ -like hypotheses, Λ_μ^e , in combination with the reconstructed momentum of the e-like hypothesis, p_e . The FV for the T2K analysis is determined using two geometric variables: the distance from the reconstructed interaction vertex to the nearest point on the inner detector wall (D_{wall}), and the distance from the vertex to the detector wall along the reconstructed particle direction (T_{owall}). Both quantities were previously introduced and discussed in Section 4.3.1 on page 83.

The D_{wall} and T_{owall} criteria are optimized separately for each sample to maximize the overall sensitivity to θ_{23} and δ_{CP} . For the μ -like samples, a minimum distance of $D_{wall} > 50$ cm and $T_{owall} > 250$ cm is required. For the e-like samples without a decay electron, the requirements are $D_{wall} > 80$ cm and $T_{owall} > 170$ cm, while for the e-like sample containing one decay electron the cuts are $D_{wall} > 80$ cm and $T_{owall} > 270$ cm.

For events with a single reconstructed track, the next step in the selection process involves particle identification (PID) between the e and μ hypotheses. Since the Cherenkov ring topologies of electrons and muons differ significantly (see Section 3.3 on page 54), this discrimination is achieved using the e/μ likelihood variable. The distribution of this PID discriminator, which serves as a key selection criterion, is shown in Fig. 4.10.

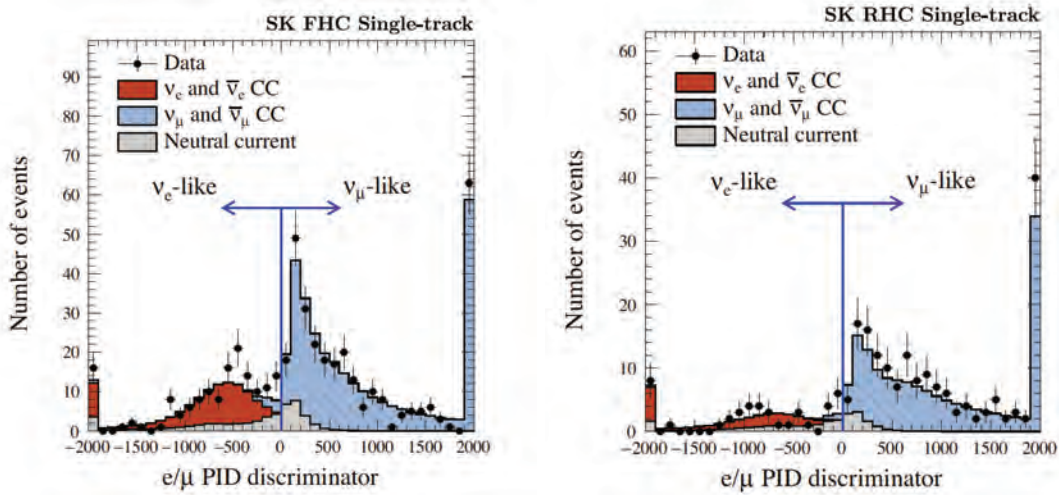


Figure 4.10. Distribution of the e/μ PID discriminator for single-ring samples in FHC (left panel) and RHC (right panel) T2K data. The figure is taken from [156].

An additional selection criterion utilizes the reconstructed momentum of the Cherenkov ring to further improve sample purity. For the e-like samples, the reconstructed momentum, p_e , is required to exceed 100 MeV/c, effectively suppressing background events originating from sub-threshold muon decays. In the case of μ -like samples, a stricter requirement of $p_\mu > 200$ MeV/c is applied to remove low-momentum events with poor reconstruction quality.

Moreover, events in the μ -like samples are allowed to contain up to one reconstructed decay electron, while the e-like samples are required to have zero or one decay electron for the selections targeting CCQE and CC1 π^+ interactions, respectively. A significant source of background in both the e-like and μ -like samples arises from neutral-current (NC) π production processes. In the case of the e-like sample, electromagnetic showers produced by $\pi^0 \rightarrow \gamma\gamma$ decays can mimic an e-like event, particularly when the two photon-induced Cherenkov rings overlap, leading FITQUN to reconstruct only a single e-like ring.

To suppress background events originating from NC π^0 interactions, a dedicated rejection criterion is applied based on the likelihood ratio between the best-fit π^0 and e-like hypotheses, $\Lambda_e^{\pi^0}$, together with

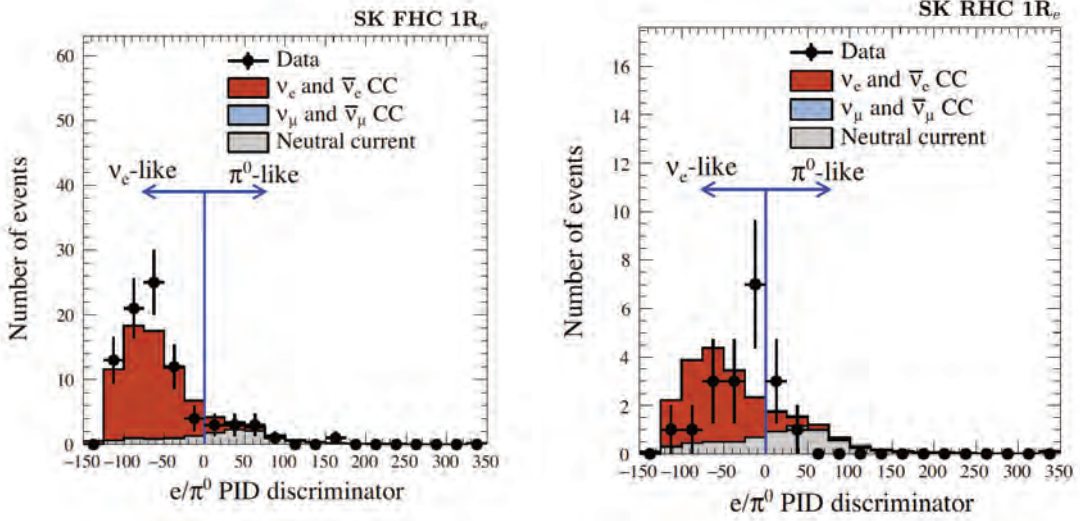


Figure 4.11. Distribution of the e/π^0 PID discriminator for 1Re sample in FHC (*left panel*) and RHC (*right panel*) T2K data. The figure is taken from [156].

the reconstructed invariant mass of the π^0 candidate, $m_{\gamma\gamma}$. The distributions of the $\text{NC}\pi^0$ rejection discriminator for the 1Re sample are shown in Fig. 4.11, separately for the T2K data collected in FHC (*left panel*) and RHC (*right panel*) beam modes.

For the μ -like samples, π^+ interactions can only be distinguished from muons through their hadronic interactions, which are not always seen in the detector. To reduce contamination from such events, a selection cut is applied on the likelihood ratio between the best-fit single- π^+ and single- μ hypotheses, $\Lambda_{\mu}^{\pi^+}$, as a function of the reconstructed muon momentum, p_{μ} . This selection criterion is implemented exclusively within the FITQUN reconstruction algorithm and was used for the first time in the results published in [156]. The distributions of the $\text{NC}\pi^+$ rejection discriminator for the 1R μ sample are shown in Fig. 4.12, for both T2K data-taking modes — FHC (*left panel*) and RHC (*right panel*).

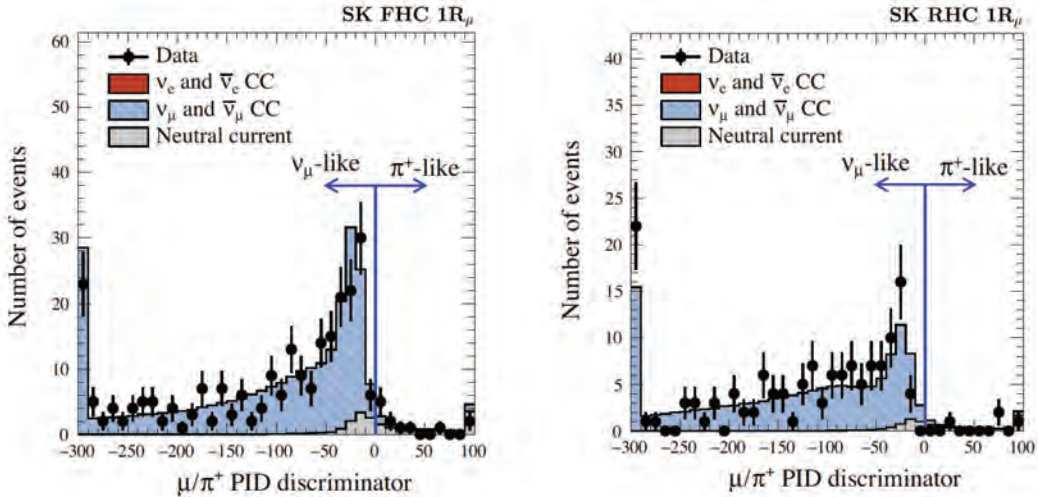


Figure 4.12. Distribution of the μ/π^+ PID discriminator for 1R μ sample in FHC (*left panel*) and RHC (*right panel*) T2K data. The figure is taken from [156].

Finally, events in the e-like samples are required to have a reconstructed neutrino energy, E_{rec} , below 1250 MeV. This requirement retains nearly all oscillation-induced ν_e signal events, which predominantly populate the low-energy region, while effectively rejecting a large fraction of the intrinsic

Table 4.3. Summary of the selection criteria for the five T2K FC, single-ring samples. These selections are used in the far detector of the T2K, SK. As it was described in the text, we use the same selections for 1R μ and 1Re in FHC and RHC beam modes. Sample 1Re + 1d.e. is only used in FHC beam mode. Taken from [156].

Selections on:	1R μ	1Re	1Re + 1d.e. (FHC mode only)
D_{wall}	> 50 cm	> 80 cm	> 50 cm
T_{owall}	> 250 cm	> 170 cm	> 270 cm
e/ μ identification	$\Lambda_{\mu}^e < \frac{p_e}{5\text{MeV}/c}$	$\Lambda_{\mu}^e > \frac{p_e}{5\text{MeV}/c}$	$\Lambda_{\mu}^e > \frac{p_e}{5\text{MeV}/c}$
e/ μ momentum	> 200 MeV/c	> 100 MeV/c	> 100 MeV/c
Number of decay -e	≤ 1	0	1
NC π rejection	$\Lambda_{\mu}^{\pi^+} < \frac{6p_{\mu}}{40\text{MeV}/c}$	$\Lambda_e^{\pi^0} < 175 - \frac{35m_{\gamma\gamma}}{40\text{MeV}/c}$	$\Lambda_e^{\pi^0} < 175 - \frac{35m_{\gamma\gamma}}{40\text{MeV}/c}$
E_{QE}^{Rec}	-	< 1250 MeV	< 1250 MeV

beam ν_e background, which tends to occur at higher energies. A summary of the SK selection criteria for the five T2K FC single-ring samples is presented in Table 4.3.

4.4. Neutrino interaction models and related systematic uncertainties

The SK atmospheric neutrino sample spans a wide range of energies and includes FC, PC, and upward-going muon (Up- μ) events. In contrast, all events in T2K are FC, and the neutrino flux has a relatively narrow energy distribution peaking around 0.6 GeV. This flux is dominated by CCQE interactions and benefits from the ND280 constraint on the product of flux and cross-section, resulting in a well-constrained neutrino interaction model. Moreover, because the true neutrino direction is known in T2K, the scattering angle can be reconstructed precisely, which is essential for calculating the reconstructed neutrino energy used in the oscillation analysis. Consequently, a dedicated systematic uncertainty model has been developed for this specific energy region, accounting for effects that can influence the reconstructed neutrino energy through uncertainties in both the reconstructed momentum and scattering angle.

For the joint fit, a common neutrino interaction model was developed where possible to ensure consistency between the two experiments. A comparable neutrino energy range is shared between the Sub-GeV SK atmospheric samples and all T2K samples. Fig. 4.13 shows the area-normalized true

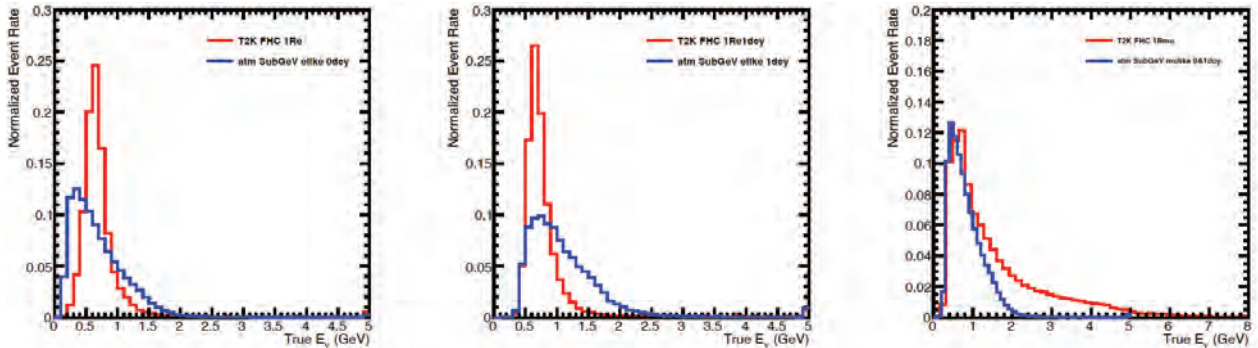


Figure 4.13. Comparison of the normalized true energy distribution for the T2K and SK atmospheric Sub-GeV samples. *Left panel:* single ring e-like events 0 dcy from SK (blue line) and 1Re events from T2K (red line); *middle panel* single ring e-like events 1 dcy from SK (blue line) and 1Re + 1d.e. events from T2K (red line); *right panel* single ring μ -like events 0 and 1 dcy from SK (blue line) and 1R μ events from T2K (red line). The figure is taken from [157].

neutrino energy distributions for the three T2K beam FHC single-ring event samples alongside the corresponding SK atmospheric samples. Although these samples cover similar energy ranges, their spectral shapes differ due to variations in the underlying fluxes and differences in the event selection criteria applied in each experiment.

For event samples that share similar energy distributions and interaction components, a common neutrino interaction model should be employed. In contrast, the SK atmospheric samples span a much broader energy range, encompassing higher-energy interactions such as deep inelastic scattering (DIS), which dominate at Multi-GeV energies and are also crucial for oscillation parameter sensitivity. Moreover, unlike T2K, the SK atmospheric data lack a near detector to constrain the unoscillated flux, and the true neutrino direction is not known, preventing the use of reconstructed scattering angle information. Consequently, the interaction model for atmospheric neutrinos is developed with an emphasis on accurately describing high-energy processes. Based on these considerations, all samples are categorized into two classes:

- low energy samples, which include all T2K samples and SK atmospheric Sub-GeV samples
- high-energy samples, which include all the rest high-energy SK atmospheric samples.

For joint analysis, we therefore choose a hybrid approach, where low-energy samples use the *T2K model*, whereas the high-energy samples use the *SK model*.

For charged-current quasi-elastic (CCQE) interactions, an exception is made to the separation between the low- and high-energy cross-section models. In this case, the core interaction model is considered sufficiently well developed, and thus the *T2K model*, including the ND280 near-detector constraints, is applied to all samples, even to the atmospheric Multi-GeV events. However, for the phenomenological normalization parameters that describe different momentum transfer (Q^2) regions, the three high- Q^2 systematic parameters—most relevant at higher energies—are treated separately. These parameters are applied only to the high-energy samples as uncorrelated, unconstrained nuisance parameters without near-detector constraints. This approach prevents over-constraining these high- Q^2 parameters using the subdominant phase-space regions probed by the near detector or by the statistically dominant low-energy samples [158].

4.4.1. Construction of the coherent model for low energy samples

As a first step toward developing a coherent interaction model across the samples of both experiments, it was decided to generate MC simulations using identical software versions and configuration settings. The configuration adopted corresponds to that used in the T2K Run 1–10 analysis, referred to here as the *T2K model*, and a new SK–IV MC sample was produced using these settings. Relative to the atmospheric MC employed in the SK–IV oscillation analysis with FITQUN [153], no modifications were made to the flux generation, and only minor adjustments were introduced to the NEUT settings governing neutrino interaction generation. Two main differences arise in the interaction model: the treatment of CCQE and 2p2h interactions. For CCQE events, the Spectral Function model is used in the combined SK+T2K framework, replacing the Local Fermi Gas (LFG) model used in the standalone SK atmospheric analysis. For 2p2h interactions, the same model as before is retained, but cross sections are now taken from precomputed tables rather than being directly evaluated from the hadron tensor.

Since T2K employs ND280 constraints on the cross-section parameters, the common interaction model for the low-energy samples was built upon the *T2K model*. Within this framework, it was natural to apply the ND280-derived constraints to the cross-section parameters relevant for the SK atmospheric Sub-GeV samples, particularly those governing CCQE interactions.

Description of the T2K model. The results presented in [46] primarily rely on the $CC0\pi$ event topology, which receives dominant contributions from CCQE, 2p2h, and $CC1\pi^+$ interactions in which the pion is either below the Cherenkov threshold, absorbed through final-state interactions (FSI), or removed via secondary interactions (SI). In the following section, we summarize the main features of the *T2K model*.

- **CCQE interactions.**

T2K models CCQE interactions using the Benhar Spectral Function (SF) approach [159], which describes the probability of finding a nucleon with a given removal energy and momentum, thereby incorporating nuclear shell structure effects. This represents a significant improvement in realism compared to the commonly used global and local relativistic Fermi gas models (RFG and LFG, respectively). The single-nucleon cross section is calculated using the BBBA05 vector form factors [160] and a dipole form factor for the axial current. The axial mass parameter in the dipole form, M_A^{QE} , is tuned to bubble chamber CCQE data from ANL [161, 162], BNL [163], BEBC [164], and FNAL [165]. Uncertainties are conservatively enlarged to encompass both the T2K fit results and the global averages [166], yielding a final value of $M_A^{\text{QE}} = 1.03 \pm 0.06 \text{ GeV}/c$.

- **2p2h interactions.**

For 2p2h interactions, T2K employs the Nieves *et al.* (Valencia) model [118], implemented in NEUT using pre-calculated lookup tables. While the underlying theoretical model is identical to that used by the SK collaboration, the treatment of systematic uncertainties differs. The uncertainty parameterization includes three independent normalization parameters corresponding to ν , $\bar{\nu}$, and $^{12}\text{C} \rightarrow ^{16}\text{O}$ interactions. The normalization parameters for ν and $\bar{\nu}$ are left completely unconstrained, while the oxygen-to-carbon scaling parameter carries a 30% prior uncertainty, with no correlations assumed between the three. In addition, T2K introduced a shape-like 2p2h uncertainty derived from comparisons between the Δ and non- Δ contributions in the Nieves model.

- **Single pion production.** Single-pion production is modeled using the same approach as in SK– (see Section 3.2.2 on page 49). The Rein–Sehgal model [119] is employed, incorporating updated nucleon form factors with an emphasis on the $\Delta(1232)$ resonance, which dominates at T2K energies [121]. Lepton mass effects are included following [120, 167]. The initial-state nuclear model assumes a global relativistic Fermi gas, while coherent pion production is treated using the Berger–Sehgal model [168]. The uncertainty parameterization is defined in terms of the free-nucleon parameters M_A^{RES} , C_5^A , and the non-resonant $I_{1/2}$ background normalization, consistent with the treatment used by SK.

- **Final state interactions.** Pions and nucleons produced in the primary neutrino interaction are propagated through the nuclear medium using an intranuclear cascade simulation. For pions, T2K employs the Salcedo–Oset model [169, 170], tuned to modern pion–nucleus scattering data [171]. The associated uncertainties are derived directly from this tuning procedure, including correlations among model parameters. For nucleons, the Bertini cascade [100] is used instead. No dedicated uncertainty model is applied in this case, as nucleon re-scattering within the nucleus has been shown to have a negligible impact on T2K analyses to date.

- **Other interactions and uncertainties.** T2K assigns a 100% normalization uncertainty to neutral-current (NC) interactions and applies an overall weighting factor of two to all such interactions to align the NEUT predictions with modern theoretical calculations. The combined uncertainty on the total NC cross section, encompassing NC elastic, NC resonant kaon and eta production, as well as NC deep inelastic scattering (DIS) processes, is taken to be 30%. These uncertainties are treated as completely uncorrelated between the near and far detectors.

The atmospheric Sub-GeV samples, however, also include several event categories that are dominated by interaction types or regions of phase space for which no dedicated near-detector (ND280) selections exist. These include the `SubGeV_ellike_1dcy` and `SubGeV_muellike_2dcy` samples, which are enriched in the low- p_π phase space of $\text{CC}1\pi^+$ interactions by requiring the presence of a decay electron but no secondary ring from the π^+ . Another example is the `SubGeV_pi0like` sample, which is enhanced in $\text{NC}1\pi^0$ interactions. Although the near-detector fit does contain $\text{CC}\nu_\mu$ interactions with additional $\pi^\pm$ tagging, its sensitivity to the pion momentum is limited, since it combines three different tagging algorithms: Michel-electron tagging for low-momentum pions ($p_\pi < 160$ MeV), FGD-contained pions for intermediate momenta ($p_\pi \in [150, 250]$ MeV) in the backward region, and TPC-tagged pions for higher-momentum pions ($p_\pi > 150$ MeV) in the forward direction, with the distribution peaking around 300 MeV and extending up to several GeV. By combining these three tagging algorithms, the overall tagging efficiency becomes relatively flat as a function of pion momentum. Consequently, the near-detector sample selection, which relies primarily on pion multiplicity, is effectively insensitive to the detailed pion momentum distribution. As a result, although the muon kinematics are well constrained by the near-detector fit, this does not translate into an equally precise prediction of the pion kinematics. In fact, the interaction model employed at the near detector lacks explicit systematic parameters that can modify the pion momentum spectrum. Therefore, to correctly extrapolate the near-detector fit results to the atmospheric $\text{CC}1\pi^+$ -enhanced samples, it is necessary to introduce an additional systematic degree of freedom associated with the pion momentum, to model effects not constrained by the near-detector data. When examining the down-going atmospheric samples at SK, which are largely insensitive to neutrino oscillations for the $\text{CC}1\pi^+$ -enhanced samples, the application of the ND280 constraint leads to a significant under-prediction of the data (Fig. 4.14). This clear excess of data events observed in the charged-current single charged pion ($\text{CC}1\pi^+$ -enhanced) samples is as follows: in the e-like sample, 225 events are observed compared to 160 ± 12.6 (stat.) ± 14.3 (syst.) predicted using the ND280 constraint, and in the μ -like sample, 84 events are observed for a prediction of 52.0 ± 7.2 (stat.) ± 5.3 (syst.). As a consequence of the observed excesses, an additional interaction

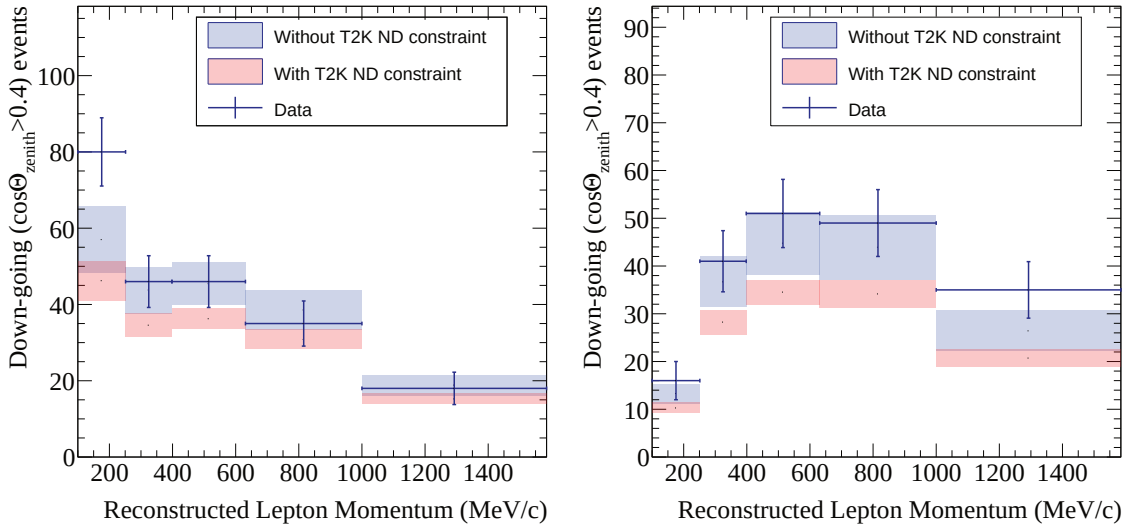


Figure 4.14. $\text{CC}1\pi^+$ -enhanced Sub-GeV down-going atmospheric samples for e-like 1 decay electron (*left panel*) and μ -like 2 decay electron sample (*right panel*) at SK. The black histogram shows the data with statistical errors. The error bands show the prior predictive error bands for atmospheric flux and interaction systematics using the *T2K model* (i.e. without introducing additional systematics for the joint fit), before (red) and after (blue) applying the T2K ND constraint. For the μ -like samples oscillations only have a limited effect on these events at high enough energy (above 631 MeV), but are applied to the predictions. The figure is taken from [158].

model uncertainty was introduced to modify the shape of the pion three-momentum spectrum in the charged-current resonant interactions. This was achieved by adjusting the Adler angle [172] distribution, following both theoretical considerations [173] and empirical modifications motivated by experimental observations [128].

4.4.2. Construction of the model for high-energy samples

The high-energy samples are all SK atmospheric samples including, FC Multi-GeV, PC and Up- μ samples. Since they are all SK samples, the systematic uncertainties are based on the *SK model* but some modifications have been made for this joint analysis:

- **CCQE interactions.** The SK atmospheric analysis uses the Valencia Local Fermi Gas model, while in the joint analysis the MC is generated by the Spectral Function (SF) model. In this first joint fit analysis, the high-energy CCQE samples use the same uncertainties as low energy CCQE samples.
- **2p2h interactions.** 2p2h interactions were not found to have a large effect on the SK Multi-GeV samples. The model is based on the *SK model* with an additional shape uncertainty to scale the fraction of 2p2h interactions.
- **Resonant interactions.** The high-energy samples are modeled using the SK framework without applying the ND constraint, since the resonance contributions affect low- and high-energy samples differently. Three key parameters govern the treatment of resonant interactions: the axial mass uncertainty M_A^{RES} , the normalization factor C_A^{RES} , and the isospin background scaling parameter $I_{1/2}$ —all of which are also included in the *T2K model*. In this joint analysis, the priors for these parameters have been updated to the values used in the T2K Run 1–10 analysis to ensure consistency, and these updates have likewise been applied in the SK atmospheric analysis.
- **Deep inelastic interactions.** The *T2K model* includes two deep inelastic modes, separated by the invariant mass. Both modes rely on the calculation of the differential cross section in Bjorken x and y , which together fully describe the inclusive DIS interaction kinematics. In the quark-parton model, this differential cross section can be computed using the parton distribution functions determined experimentally. The GRV98 PDFs [125], modified by Bodek and Yang [174] are used. DIS uncertainties include both the Bodek–Yang corrections and the total cross section. This interaction model is well developed in the SK atmospheric analysis, and the joint fit uses the DIS model based on the *SK model*. Key differences between the high-energy and low-energy samples include: uncertainty on the Bodek–Yang model for multipion events is separated for the axial and vector part of the structure functions with an additional 3% normalization uncertainty, and an additional uncertainty from the comparison of the nominal model to an alternative model called CKMT[141].

4.4.3. Other sources of systematic uncertainties relevant for the joint SK+T2K oscillation analysis

In the independent analyses published by SK [153] and T2K [46], the parameters describing systematic uncertainties were treated separately, even though both analyses use the same water Cherenkov detector. As discussed in Section 4.3 on page 83, SK and T2K use different event selection and classification procedures. Furthermore, the SK analysis does not use near-detector data to constrain the atmospheric flux and cross-section uncertainties. Additional distinctions arise from the different energy ranges of the two experiments, with SK studying a broader and higher-energy spectrum of neutrinos. Consequently, some components of the systematic uncertainties are independent between the two experiments and required reevaluation for a coherent joint analysis. In this work, a unified framework was constructed in which the systematic uncertainties are consistently organized into three major categories: neutrino

flux predictions, cross-section parameters (as detailed in the previous sections), and detector-related effects.

Neutrino flux predictions As discussed in the previous chapters, the two experiments study neutrino oscillations using neutrinos originating from different sources and use distinct models to predict the corresponding neutrino fluxes. In T2K (see Subsection 2.3.2 on page 40), the beam flux prediction is based on a series of simulations tuned to external hadron-production data, primarily from the NA61/SHINE experiment. In contrast, the SK atmospheric flux model (see Subsection 3.2.1 on page 48) is derived from the Honda flux calculations, which incorporate measurements from experiments such as BESS and AMS for the primary cosmic-ray flux. The two flux models, therefore, rely on different physical inputs, treat independent sources of uncertainty, and produce systematic errors on different quantities. We have therefore concluded that correlating the flux uncertainties between the two experiments is not currently justified. For the joint fit, each experiment’s flux model is used independently, with no correlations included between the flux uncertainties of the two models. This approach is considered robust, given the substantial differences in the underlying flux sources, modeling techniques, and dominant error contributions between the beam and atmospheric neutrino flux predictions. However, we note that work on the main atmospheric flux models (Honda and Bartol) is on going intending to use also tuning from external hadron production measurements, like NA61/SHINE data, as it was used in T2K. Once those updates are available, a more robust treatment of the SK and T2K flux uncertainties, including correlations between hadron production errors, will likely be possible and even necessary. However, this is expected to happen on a longer timescale.

Detector systematics Many of the detector-related uncertainties in the SK and T2K analyses are evaluated through comparisons between atmospheric neutrino data and corresponding MC simulations. For these uncertainties, correlated components are constructed by simultaneously evaluating the effects of detector parameter variations on the event rates in both the SK and T2K samples. Correlations between the reconstructed momentum scale uncertainties of the two experiments are found to have a notable impact on the Δm_{32}^2 constraint obtained from the joint data fit. Other detector uncertainties from the reference analyses that are relevant to only one of the two experiments are applied exclusively to the corresponding samples in this work. In addition, an extra systematic uncertainty is introduced for the Sub-GeV samples targeting $\text{CC}1\pi^+$ events. This uncertainty accounts for possible event migration between the ν_e -like and ν_μ -like samples arising from the single-ring, single Michel-electron events with a low lepton momentum. Its magnitude is chosen to encompass the observed excess of down-going $\text{CC}1\pi^+$ events at low momentum in the data.

4.5. Results on oscillation analysis

The results presented in this section were obtained using four independent analysis frameworks [152]: two Bayesian (BA1 and BA2) and two frequentist (FA1 and FA2). The Bayesian frameworks, BA1 and BA2, are based on the T2K analysis framework, while FA1 represents an adaptation of BA1 designed to render frequentist analyses computationally efficient. The second frequentist framework, FA2, originates from the SK analysis chain and serves both as an independent validation of the SK implementation within the T2K-based frameworks and as a consistency cross-check of the overall analysis methodology.

The Bayesian analyses (BA1 and BA2) employ Markov Chain Monte Carlo (MCMC) sampling to estimate the posterior distributions and corresponding marginal likelihoods of the oscillation parameters. Conversely, the frequentist frameworks (FA1 and FA2) evaluate profile likelihoods across a fixed grid in the oscillation parameter space. All four analyses make use of a binned negative log-likelihood test

statistic constructed under the assumption of Poisson-distributed event counts, with additional Gaussian penalty terms applied to constrain systematic parameters around their nominal values. A comprehensive description of the statistical methodologies and implementation details can be found in [152].

We use the reactor experiment measurements of θ_{13} using $\bar{\nu}_e$ disappearance, $\sin^2 2\theta_{13} = 0.0853 \pm 0.0027$ [175] as an external constraint.

4.5.1. Bayesian results

Historically, investigations of CP violation in neutrino oscillation experiments within a Bayesian framework have primarily focused on assessing whether the specific hypotheses $\delta_{\text{CP}} = 0$ or π can be excluded from the credible intervals, either assuming a fixed neutrino mass ordering or after marginalizing over both orderings. In more recent analyses, the T2K and NO ν A collaborations have extended this approach by expressing their sensitivity in terms of the Jarlskog invariant, J , and by testing whether its value is consistent with zero. This formulation provides a parameterization-independent and physically transparent means of quantifying CP violation, allowing the results to be interpreted without dependence on the chosen phase convention.

The Bayesian analyses assume uniform priors on δ_{CP} or $\sin \delta_{\text{CP}}$, $\sin^2 \theta_{23}$, Δm_{32}^2 , and the mass ordering (MO). The results indicate a preference for the normal mass ordering and a mild preference for the upper octant (Table 4.4).

Table 4.4. Posterior probabilities for the θ_{23} octant and neutrino mass ordering, derived using either the full joint dataset or individual experimental samples, assuming equal prior probabilities. Results from the second analysis are shown in parentheses.

	SK only	T2K only	SK + T2K
Upper octant	0.318 (0.337)	0.785 (0.761)	0.611 (0.639)
Normal ordering	0.654 (0.633)	0.832 (0.822)	0.900 (0.887)

Differences between SK and T2K data in their preferred octants lead the combined analysis to exhibit a weaker octant constraint than either experiment individually, while simultaneously showing a stronger tendency toward maximal mixing. Both experiments favor similar values of the CP-violating phase (Fig. 4.15, *left panel*).

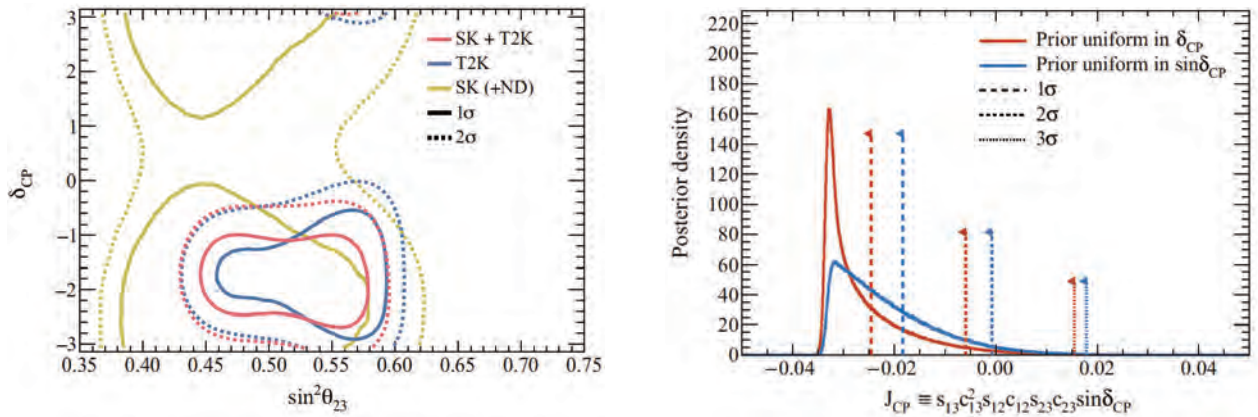


Figure 4.15. *Left panel:* Credible regions in the $(\sin^2 \theta_{23}, \delta_{\text{CP}})$ plane obtained from the SK, T2K, and combined datasets. The mass ordering is marginalized over, and a prior uniform in δ_{CP} is assumed. *Right panel:* Posterior probability density of the Jarlskog invariant, J_{CP} , with credible intervals overlaid. The distribution is marginalized over both mass orderings and computed assuming a uniform prior in either δ_{CP} or $\sin \delta_{\text{CP}}$. The left edges of the credible intervals nearly coincide in the region $-0.035 \leq J_{\text{CP}} \leq -0.033$. The figures are taken from [152].

Differences between the preferred octants in the SK and T2K data lead the combined analysis to exhibit a weaker constraint on the θ_{23} octant than either experiment individually, while simultaneously showing a stronger preference for maximal mixing. Both experiments, however, favor similar values of the CP-violating phase (see Fig. 4.15, *(left panel)*).

The degree to which CP-conserving values of J_{CP} (Fig. 4.15, *(right panel)*) and δ_{CP} are disfavored, is evaluated as the maximum fraction of the posterior probability density from which these values are excluded within the highest-posterior-density credible intervals obtained in either of the two Bayesian analyses.

4.5.2. Frequentist results

As discussed in Subsection 3.6.4 on page 75, Wilks' theorem [150] often used for constructing confidence intervals (i.e. $\Delta\chi^2 < 1$ for 1σ confidence interval on 1D parameter θ etc.) is only asymptotically valid for unbounded continuous degrees of freedom. For the oscillation parameters neither condition is satisfied: the mixing angle $\sin^2 \theta_{23}$ and δ_{CP} are angular variables and hence circular and bounded and the mass ordering acts as a discrete parameter. To construct proper frequentist confidence intervals we therefore need to explicitly correct for the coverage by fitting ensembles of toy MC experiment instead of relying on Wilks' theorem. On the other hand, if the mass ordering is fixed, Δm_{32}^2 is expected to satisfy the conditions required by Wilks' theorem, which will be used as a way to validate the proposed method. The Feldman-Cousins method [176] is often employed for the construction of confidence intervals with proper coverage.

In the case of neutrino mass ordering (MO) we have two hypotheses: normal (NO) and inverted (IO) ordering. The posterior distributions of the oscillation parameters under two neutrino mass hypotheses can be found in Fig. 4.16. These distributions are used to construct toy experiments, based on pseudo data sets generated from the T2K neutrino MC. For each neutrino mass ordering (MO) hypotheses, several thousand toy experiments were generated, and fits to both assumptions were performed. Thus, the difference in the best-fit values χ^2 in both assumptions can be obtained ($\chi_{\text{NO}}^2 - \chi_{\text{IO}}^2$).

The distribution of this MO test statistic can be found in the *(left panel)* in Fig. 4.17. For the T2K data p -value is equal to 0.08. In case of the toy experiments we have obtained the p -value for normal mass ordering equal to 0.58, while the p -value for inverted mass ordering was 0.077. For the discovery announcement, the CL_s is performed to show the sufficiency of the statement. The CL_s can be computed using Eq. (3.31) on page 76.

As a result of the present studies, a mild preference for the normal mass ordering is observed, corresponding to a 1.2σ exclusion of the inverted ordering.

To test for CP conservation (CPC), toy MC experiments are generated for both MO assuming either $\delta_{\text{CP}} = 0$ or π . A test statistic for CPC is given by $\Delta\chi^2 = \chi^2(\text{CPC}) - \chi^2(\text{CPV})$, where CPV stands for CP validated, and $\chi^2(\text{CPV})$ represents the best fit to toy MC in both MO, and $\chi^2(\text{CPC})$ is the minimum value χ^2 among the following four cases: $(\delta_{\text{CP}} = 0, \text{NO})$, $(\delta_{\text{CP}} = 0, \text{IO})$, $(\delta_{\text{CP}} = \pi, \text{NO})$, $(\delta_{\text{CP}} = \pi, \text{IO})$.

Several thousands toy experiments have been generated, and the distribution of the CPC test statistic is shown in the *(right panel)* in Fig. 4.17. The $\Delta\chi^2 = \chi^2(\text{CPC}) - \chi^2(\text{CPV})$ from the data is 3.78, which leads to a CPC p -value of 0.030.

4.6. Discussion and conclusions

In this chapter, we have presented a data-driven results using SK atmospheric data from the SK-IV period (3244.4 days, corresponding to 264.2 kt-years of exposure) and T2K Run 1–10 data (1.9664×10^{21} protons on target [POT] in neutrino mode and 1.6346×10^{21} POT in antineutrino mode).

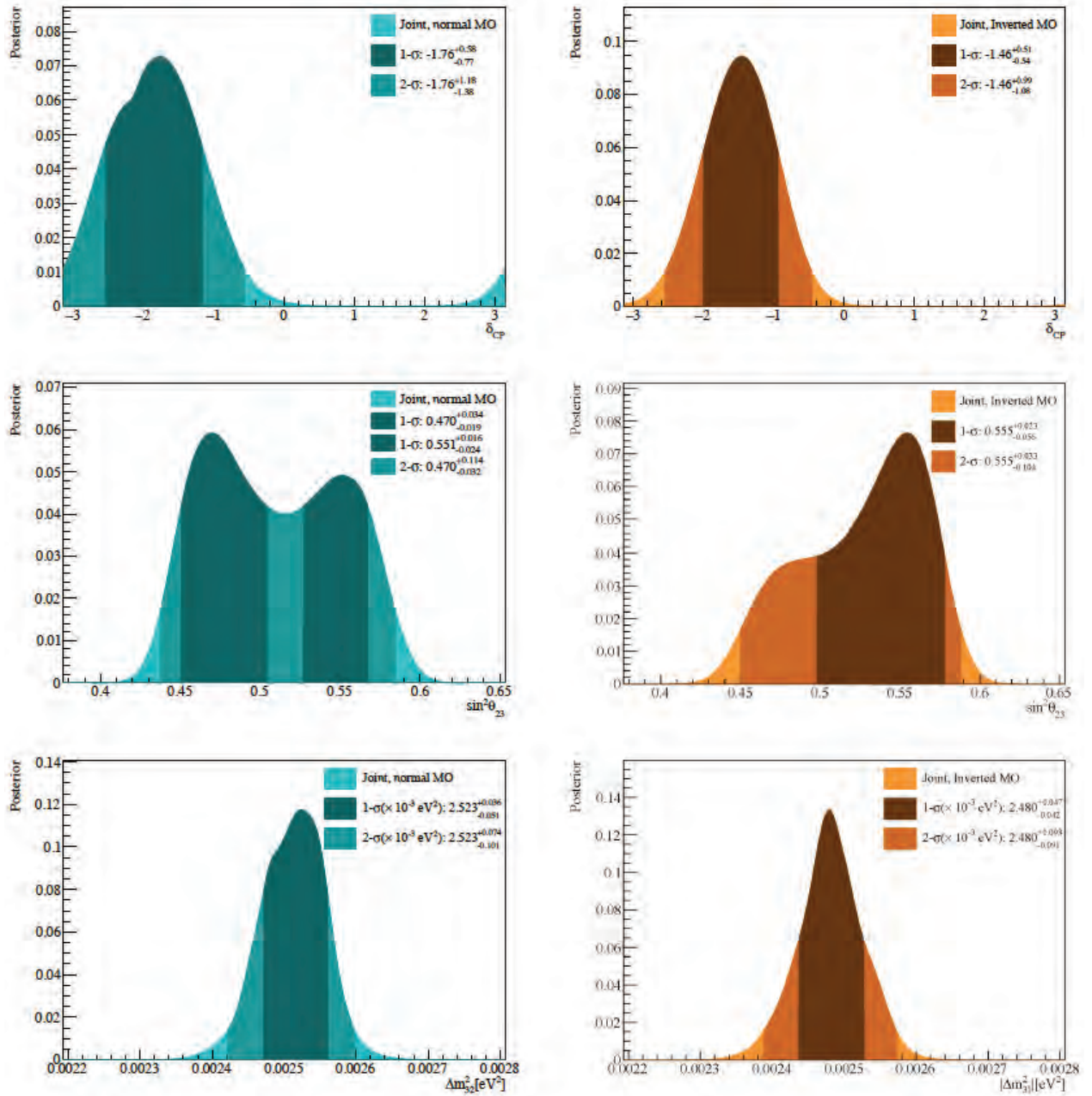


Figure 4.16. The posterior distributions of the oscillation parameters for two neutrino mass orderings. The figure is taken from [158].

The SK and T2K datasets indicate similar preferred values for the CP phase, suggesting nearly maximal CP violation, and both exhibit a preference for the normal mass ordering (MO). Consequently, the combined analysis strengthens the indication of CP non-conservation. Examining the direct exclusion of CP conservation, through the presence of $J_{CP} = 0$ within credible intervals or Frequentist p -values, yield an exclusion at the 2σ level. However, the significance may drop below 2σ due to potential limitations in the datasets.

The SK and T2K collaborations have completed their first combined analysis of the two datasets. To achieve this, unified models for neutrino interactions and detector responses were developed to cover the overlapping energy regions, and these models successfully describe both sets of data. The analysis excludes the CP-conserving value of the Jarlskog invariant with a significance ranging from

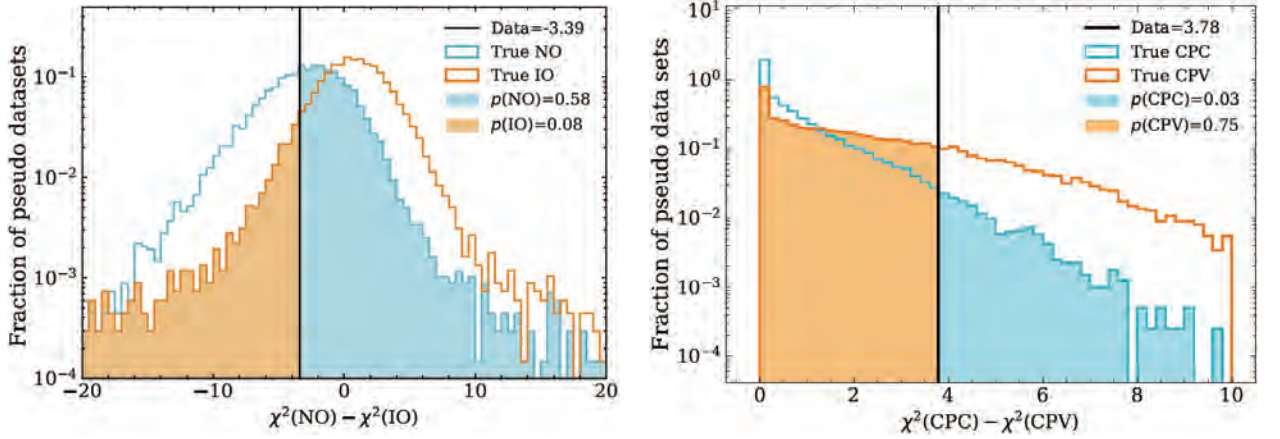


Figure 4.17. *Left panel:* Distribution of the mass ordering (MO) test statistic for the normal (NO) and inverted (IO) hypotheses. Shaded areas indicate the p -values corresponding to the alternative ordering relative to the observed data. *Right panel:* Distribution of CPC test statistic under true CPC hypotheses discussed in the text. The figures are taken from [152, 158].

1.9σ to 2.0σ . It also shows a slight preference for the normal mass ordering, with the inverted ordering disfavored at 1.2σ , while no decisive evidence is found for the θ_{23} octant.

Moreover, the presented joint fit is the first performed collaboratively by both experiments: SK and T2K and represents an important milestone toward the success of the next-generation Hyper-Kamiokande experiment [50], which features an eightfold increase in fiducial volume and 2.6 times higher beam power [177]. Hyper-Kamiokande (see next chapter) aims for the first 5σ discovery of CP violation in the leptonic sector and confirmation of the neutrino mass ordering, which by that time is expected to be determined at roughly 3σ by other experiments. The experiment will integrate the atmospheric and long-baseline accelerator programs into a single collaboration, simplifying future joint fits and enabling the development of coherent interaction and detector systematic models. Several upgrades to the near detectors are ongoing [178], alongside efforts to further reduce atmospheric flux uncertainties. Combined with the substantial increase in statistics, these improvements are expected to enhance the impact of near detector constraints on beam flux and interaction model systematics in future joint analyses.

Future searches of GeV neutrino oscillations: Hyper-Kamiokande project

In this chapter, we will describe the expected sensitivity to the neutrino oscillation parameters of the future Hyper-Kamiokande (HK) long-baseline program [51]. Tokai to Hyper-Kamiokande (T2HK) experiment, currently under construction in Japan, will measure the oscillations of accelerator-produced neutrinos with thousands of selected events per sample: this corresponds to an increase in statistics of a factor 25 to 100 with respect to recent results from the currently running long-baseline neutrino oscillation experiment in Japan, T2K. Future analyses by HK will be performed with access in the same experiment to both accelerator beam data and atmospheric neutrinos. This combination, as greatly described in Chapter 4 by the joint analysis done by the T2K and SK collaborations [152], will allow HK to break the degeneracies between δ_{CP} and neutrino mass ordering (MO).

Our current knowledge of the neutrino oscillation parameters includes the measurements on the three mixing angles, as well as the Δm_{21}^2 and Δm_{32}^2 [26]. The phase δ_{CP} is still unknown, however we already have first hints from T2K [46], which point to a large CPV, but they are not confirmed by NO ν A [179]. The sign of Δm_{32}^2 , which is known as the mass ordering, is still to be experimentally confirmed, however we have first indications from SK [139] atmospheric neutrino measurements showing a preference for normal ordering ($m_3 > m_2$), greatly described in Chapter 3. Moreover, we still don't know the octant of the θ_{23} mixing angle, which if equal to $\theta_{23} = \frac{\pi}{4}$ means we have the maximum mixing between muon and tau neutrinos.

T2HK experiment is not the only one that will address all these open questions in the neutrino oscillation physics. In the next years, these neutrino oscillation parameters will be measured by another long-baseline experiment, DUNE [52], under construction in the US.

5.1. Tokai to Hyper-Kamiokande experiment

The Tokai to Hyper-Kamiokande (T2HK) experiment consists of the existing J-PARC neutrino beam and a set of near detectors, both currently used for the T2K experiment and being upgraded to increase performance, and two new water Cherenkov detectors, an intermediate detector of about 600 tons at around 1 km from the neutrino beam production target and a 258,000 ton far detector, called Hyper-Kamiokande (HK) at 295 km (oscillation baseline).

Hyper-Kamiokande (HK) is the third-generation water Cherenkov neutrino detector in Japan, following Kamiokande [29] and Super-Kamiokande [59], the experiment is currently underway. T2HK far detector will be built 295 km from the production target and situated beneath the peak of Mt. Nijyugo, resulting in a 1,750 meters-water-equivalent overburden. The detector is a 71 m high and

68 m diameter cylinder filled with ultra-pure water and split into optically isolated inner and outer detector regions. The outer detector will consist of a 1 m wide shell surrounding the inner detector barrel (top and bottom) and is planned to be instrumented with approximately 3,600 8 cm diameter photomultiplier tubes PMTs, each equipped with a 30 cm×30 cm wavelength shifting plate to enhance light collection. The outer detector will identify and veto entering charged particles, such as cosmic muons or particles created by neutrino interactions in the surrounding rock

Alongside the upgraded ND280, a new water Cherenkov detector will be built approximately 1 km from the neutrino production target [50]. The Intermediate Water Cherenkov Detector (IWCD) preliminary design consists of an 8.8 m diameter and 10 m tall cylinder with a 7 m diameter and 8 m tall inner detector region. The inner detector will be instrumented with about 350 multi-PMT modules and will have a target mass of approximately 300 tonnes. The detector will be placed in a vertical pit allowing the detector to be positioned anywhere between 4° and 1.5° off the central axis of the neutrino beam. T2HK has a general concept very similar to the T2K experiment, and as a result many analysis techniques used in T2K will be adopted here.

The T2HK analysis is currently based on the well-established oscillation analyses performed by T2K, with modifications to take into account the novelties of the T2HK, including the upgraded T2K Near Detector and J-PARC where the neutrino beam is produced, the addition of IWCD and new, larger HK detector.

5.2. Event simulation

As stated in [51], T2HK is expected to collect statistics from 2.7×10^{21} Protons-On-Target (POT) per calendar year, corresponding to 6 cycles of 22 days with 87% running efficiency at 1.3 MW. We chose a partition of 1/4 of the exposure in neutrino (FHC) and 3/4 in antineutrinos (RHC). It was decided that way in order to have influence on controlling the systematic uncertainties and direct access to extensive statistics of both neutrino and antineutrino events for a robust assessment of CP violation.

The T2HK analysis is currently based on the well-established oscillation analyses performed by T2K, with modifications to take into account the novelties of the T2HK, including the upgraded T2K Near Detector and J-PARC, where the neutrino beam is produced, the addition of IWCD and new, larger HK detector. We start discussion of the analysis from the neutrino flux model. In T2HK this model is based on the T2K flux model [72]. The expected neutrino flux has been adjusted to account for the anticipated increase in horn current (from 250 kA, used in the T2K flux simulation, to 320 kA for T2HK) as well as the different relative position of the far detector (maintaining the same off-axis angle but located on the opposite side of the beam center). The resulting simulated flux is shown in Fig. 5.1. The initial flux uncertainties—prior to any constraints from the near detector—remain as reported in [46], benefiting from the NA61/SHINE hadroproduction measurements performed with the T2K replica target [80].

The expected event rates and kinematic distributions at the far detector are modeled using the T2K Monte Carlo simulation. For the sensitivity studies presented in this section, we assumed the value of oscillation parameters reported in Table 5.1.

Table 5.1. Values of the oscillation parameters assumed for the sensitivity studies.

$\sin^2 \theta_{12}$	Δm_{21}^2 (eV ²)	$\sin^2 \theta_{23}$	Δm_{32}^2 (eV ²)	$\sin^2 \theta_{13}$	δ_{CP} (rad)	mass ordering
0.307	7.53×10^{-5}	0.528	2.509×10^{-3}	0.0218	−1.601	normal

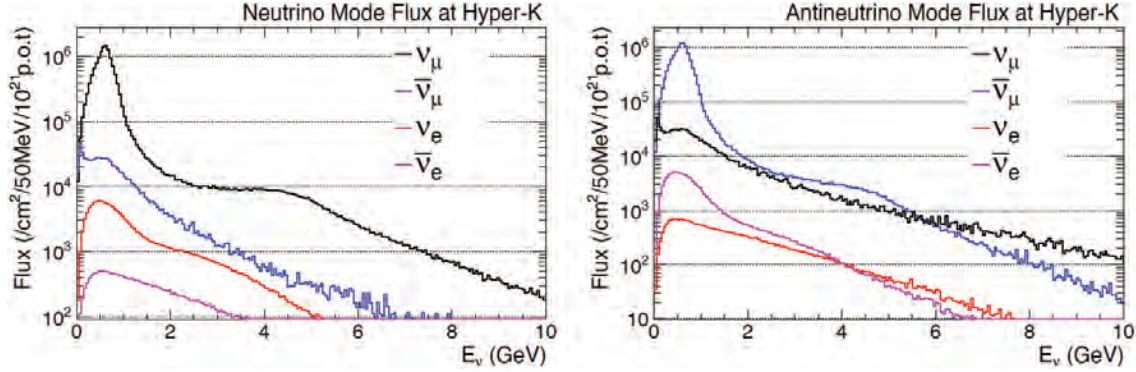


Figure 5.1. The predicted unoscillated neutrino fluxes at the far detector of T2HK in ν -mode (*left panel*) and $\bar{\nu}$ -mode (*right panel*) in the logarithmic scale with an extended E_ν range, after the tuning to NA61/SHINE data on the T2K replica target. Figure is taken from [51].

Table 5.2. Expected number of events at Hyper-Kamiokande with 27×10^{21} POT (6.75×10^{21} POT in FHC and 20.25×10^{21} POT in RHC), corresponding to 10 years of accumulated statistics. The first four columns correspond to non-oscillated events which include electron (anti)neutrinos from the intrinsic beam contamination.

	Non-oscillated beam				Oscillated signal		Total
	beam ν_μ	beam ν_e	beam $\bar{\nu}_\mu$	beam $\bar{\nu}_e$	$\nu_\mu \rightarrow \nu_e$	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	
FHC 1Re	143.9	294.3	5.3	12.0	2007.5	11.7	2474.7
RHC 1Re	59.1	130.1	96.3	234.8	229.2	793.2	1542.7
FHC 1Re + 1d.e.	14.0	40.2	0.6	0.3	255.3	0.2	310.6
FHC 1R μ	8355.4	8.4	478.0	0.7	2.6	0.01	8845.1
RHC 1R μ	4255.9	6.0	7759.9	4.7	0.2	0.4	12 027.2

Comprehensive details of the simulation, event reconstruction, and selection procedures are provided in [46] and were greatly described in Subsection 4.3.3 on page 85. A scaling has been applied to the generated Monte-Carlo events to take into account the increased size of the Hyper-Kamiokande far detector compared to Super-Kamiokande.

The expected number of events is summarized in Table 5.2, and their kinematic distributions are shown in Fig. 5.2.

The neutrino flux, neutrino–nucleus interaction cross-section, and detector response models used in this analysis are those developed by the T2K collaboration and we therefore adopt the same parameterization of the model uncertainties. A detailed description of these can be found in [180] and were also covered in Section 4.4 on page 88.

In Table 5.3 we present 1σ uncertainty on the number of events expected in each sample for each source of systematic uncertainty.

5.3. Results

The neutrino oscillation parameter sensitivities presented here are based on the full three-flavor PMNS framework of neutrino mixing. The oscillation parameters δ_{CP} , θ_{23} , and Δm_{32}^2 are fitted without external constraints, while θ_{13} is fitted both with and without a Gaussian prior of $\sin^2 2\theta_{13} = 0.0853 \pm 0.0027$ derived from reactor antineutrino measurements [26]. The parameters $\sin^2 \theta_{12} = 0.307$ and $\Delta m_{21}^2 = 7.53 \times 10^{-5} \text{eV}^2$ are fixed to the values obtained from solar and reactor experiments [26].

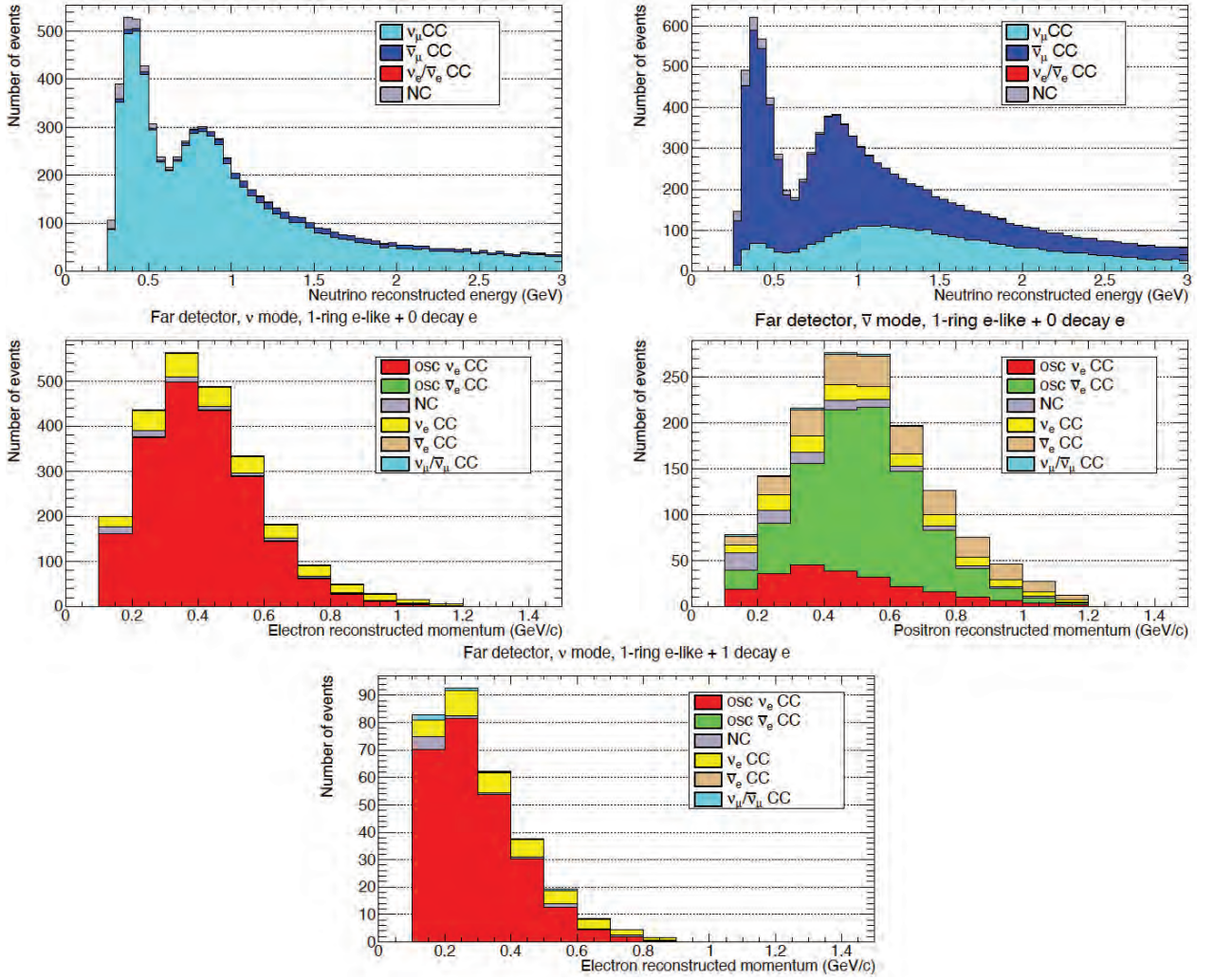


Figure 5.2. Reconstructed spectra of the selected samples are predicted for 27×10^{21} POT (6.75×10^{21} in FHC and 20.25×10^{21} in RHC), corresponding to ten years of accumulated statistics. The electron (anti)neutrino samples are divided into appearance neutrinos from oscillations (“osc”) and intrinsic electron neutrinos originating from the beam. The figure is taken from [51].

The fit results for the oscillation parameters, based on a data sample corresponding to ten years of HK exposure, are shown in Fig. 5.2 as a function of the number of electron (anti)neutrino candidates, which constitute the most sensitive samples for the CP-violation search. The best-fit prediction, along with its statistical and systematic uncertainties, is compared with the expected event distributions for different values of the oscillation parameters. Fig. 5.3 illustrates the evolution of the CP-violation discovery sensitivity with data-taking time. Under the assumption of maximal CP violation ($\delta_{\text{CP}} = -\pi/2$), T2HK is expected to reach a 5σ discovery significance in under three years.

Even under a conservative assumption on systematic uncertainties—taking the same values as in T2K [46]—T2HK is expected to reach a 5σ sensitivity to CP violation in less than six years. Given the rapid discovery potential in this scenario, the possible impact of undetermined mass ordering (MO) must be considered. Assuming the degenerate case of $\delta_{\text{CP}} = -\pi/2$ with inverted ordering, and accounting for HK MO sensitivity derived from combined atmospheric and beam data [50], the CPV discovery would be delayed to approximately six years, even when improved systematic uncertainties are assumed. This estimate is obtained by treating the MO determination from atmospheric HK data as an external constraint in the CPV analysis using beam data. It represents a conservative scenario, as

Table 5.3. 1σ uncertainty on the number of events expected in each sample for each source of uncertainty, using either the same systematic errors as the T2K analysis [180] after the near detector fit (T2K syst.) or an improved error model considering ten years of data accumulated at the near and far detectors (Impr. syst.). The uncertainty on the ratio of 1Re event samples in FHC and RHC is shown in the last column. Taken from [51].

T2K systematics	FHC 1Re	FHC 1R μ	RHC 1Re	RHC 1R μ	FHC 1Re + 1d.e.	FHC/RHC 1Re
Flux-xsec	3.6%	2.1%	4.3%	3.4%	4.9%	4.4%
Detector	3.1%	2.1%	3.9%	1.9%	13.2%	1.1%
All	4.7%	3.0%	5.9%	4.0%	14.1%	4.6%
Improved systematics	FHC 1Re	FHC 1R μ	RHC 1Re	RHC 1R μ	FHC 1Re + 1d.e.	FHC/RHC 1Re
Flux-xsec	1.8%	0.9%	1.6%	0.9%	1.8%	1.9%
Detector	1.1%	0.8%	1.5%	0.7%	4.9%	0.4%
All	2.1%	1.2%	2.2%	1.1%	5.2%	2.0%

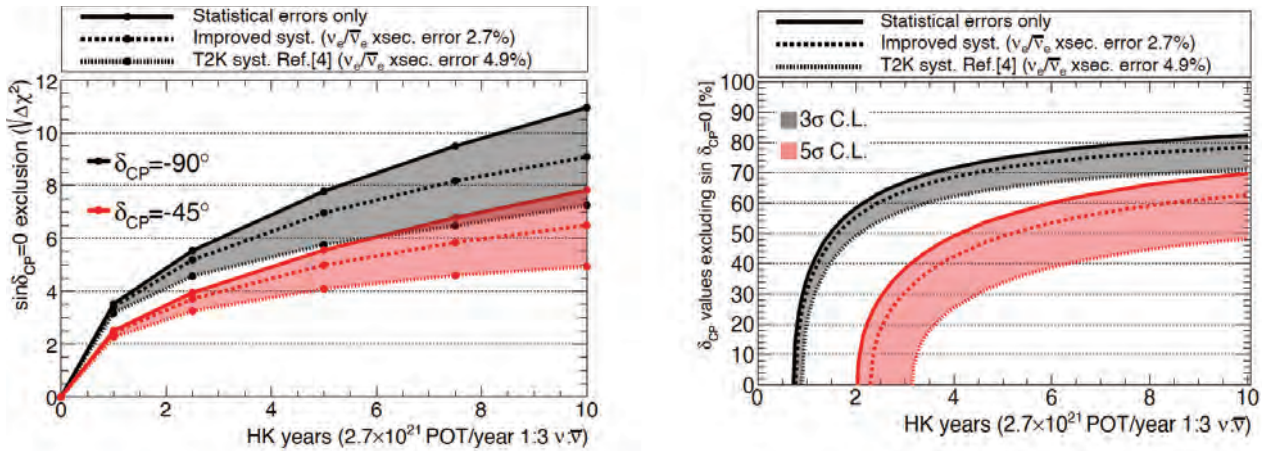


Figure 5.3. Sensitivity to CP violation as a function of data-taking time: exclusion of $\sin \delta_{CP} = 0$ for $\delta_{CP} = -90^\circ$ and -45° (left panel), and the fraction of δ_{CP} values for which $\sin \delta_{CP} = 0$ can be excluded at 3σ and 5σ (right panel). The shaded regions indicate the range of possible sensitivities arising from variations in the assumed systematic uncertainties. The figure is taken from [51].

it neglects both external MO measurements from other experiments and the enhanced CPV sensitivity achievable through a full joint fit of beam and atmospheric data within HK, as performed by the T2K and SK collaborations [152]. A dedicated joint beam–atmospheric analysis for HK is currently underway, and its sensitivity will be presented in a future publication.

5.4. Summary

In this work, we evaluate the expected sensitivity of the T2HK experiment to long-baseline neutrino oscillation parameters using accelerator (anti)neutrino beams. The results focus on the experiment’s capability to discover CP violation. Sensitivities and precision measurements are evaluated for the oscillation parameters δ_{CP} , $\sin^2 \theta_{23}$, Δm_{32}^2 , and $\sin^2 \theta_{13}$. This study builds upon the T2K analysis presented in [46], with adjustments to the neutrino flux prediction and far-detector simulation to reflect the T2HK design [50]. Various assumptions on systematic uncertainties are considered, starting from those used in T2K [46] and incorporating further reductions expected from the increased HK statistics and the upgraded ND280 and IWCD detector capabilities.

According to the assumed T2HK operating scenario, a 5σ discovery of CP violation could be achieved in under three years if maximal CP violation and a known mass ordering are assumed. Without external information on the mass ordering, and when considering the experiment's own sensitivity from atmospheric neutrino measurements, the CPV discovery is expected to require about six years. A comprehensive joint analysis combining Hyper-Kamiokande beam and atmospheric neutrino data will be reported in the next HK publication.

Summary

In this work, we have described the leading role of the Super-Kamiokande (SK) detector in neutrino oscillation measurements over the past several decades. Although SK was originally designed to search for proton decay and study solar neutrinos, it has evolved into one of the world's most important facilities for neutrino oscillation research.

The groundbreaking observation of a zenith-angle-dependent deficit of atmospheric muon neutrinos, discussed in Chapter 1, provided the first compelling evidence for neutrino oscillations and demonstrated that neutrinos possess nonzero mass. This landmark discovery earned T. Kajita and A. McDonald the 2015 Nobel Prize in Physics. Since then, SK has continued to play a central role in studying neutrino oscillations and constraining the oscillation parameters Δm_{32}^2 and $\sin^2 \theta_{23}$.

Chapter 3 of this monograph presents a measurement of the oscillation parameters $\sin^2(\theta_{23})$, Δm_{32}^2 , and δ_{CP} , as well as the determination of the neutrino mass ordering, using atmospheric neutrino data collected with the SK detector over 6511.3 live-days, corresponding to a total exposure of 484 kiloton-years [93]. This is the first SK atmospheric neutrino oscillation analysis to include fully contained (FC) events reconstructed in the expanded fiducial volume region, which the author has contributed to. The total exposure for FC events in this analysis is 484.2 kiloton-years, representing a 48 % increase over the previously published analysis [93]. An event selection based on neutron tagging was introduced to improve the statistical separation between neutrino and antineutrino interactions, thereby enhancing the sensitivity to the neutrino mass ordering. The analysis of SK data, performed with constraints on $\sin^2 \theta_{13}$, yields the oscillation parameters $\Delta m_{32}^2 = 2.40_{-0.09}^{+0.07} \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} = 0.450_{-0.026}^{+0.057}$, and $\delta_{\text{CP}} = -1.745_{-1.252}^{+0.760}$. The results show a preference for the normal mass ordering over the inverted ordering at the 92.3 % confidence level.

In the GeV energy range, SK serves not only as a detector for atmospheric neutrinos but also as the far detector for long-baseline accelerator experiments such as T2K (Tokai-to-Kamioka). In this regime, neutrinos interact primarily through charged-current quasi-elastic (CCQE) processes and single-pion production, enabling reconstruction of both neutrino energy and flavor. The excellent particle identification capabilities of SK —particularly its ability to distinguish Cherenkov rings produced by electrons and muons—allow for precise separation of ν_e and ν_μ interactions. This capability is essential for measuring appearance and disappearance probabilities, which depend on the mixing angles θ_{23} and θ_{13} , as well as for probing the CP-violating phase δ_{CP} in the PMNS matrix. The results of the first joint SK+T2K oscillation analysis [152] were presented in Chapter 4. This study used 3244.4 days of atmospheric neutrino data collected during the SK-IV period (2008–2018), corresponding to approximately 50% of the statistics employed in the analysis described in Chapter 3. The combined

SK+T2K analysis excluded the CP-conserving value of the Jarlskog invariant with a significance between 1.9σ and 2.0σ . It also showed a slight preference for the normal mass ordering, with the inverted ordering disfavored at 1.2σ , while no decisive conclusion was reached regarding the θ_{23} octant.

Furthermore, the precise event reconstruction, low background contamination, and long-term operational stability of Super-Kamiokande have established it as one of the leading detectors in neutrino physics. Its design and performance have directly influenced the development of the next-generation Hyper-Kamiokande detector in Japan. The Hyper-Kamiokande experiment will further enhance the sensitivity to leptonic CP violation and neutrino mass ordering through its substantially larger fiducial volume and improved control of systematic uncertainties. Sensitivity studies for the Tokai-to-Hyper-Kamiokande (T2HK) project are discussed in Chapter 5. According to the proposed T2HK operating scenario, a 5σ discovery of CP violation could be achieved in under three years, assuming maximal CP violation and a known mass ordering. Without external constraints on the mass ordering, and when incorporating Hyper-Kamiokande's own sensitivity from atmospheric neutrino measurements, the same discovery would require approximately six years of data taking.

We can thus look forward to an exciting future in neutrino physics, with Super-Kamiokande, which in 2026 will celebrate its 30th anniversary of successful neutrino data taking, as well as with the Hyper-Kamiokande project, which will continue to deepen our understanding of neutrino oscillations and the fundamental properties of these elusive particles.

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